

D.N.R.COLLEGE (A) BHIMAVARAM

**P.G. DEPARTMENT OF
ECONOMICS**



E-NOTES

M.A ECONOMICS

SEMESTER -II PAPER – I

MICRO ECONOMIC ANALYSIS-II

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UNIT-I

TOPIC: Oligopoly

An oligopoly market is a market condition where a few companies run the market. They can sell either homogeneous products or heterogeneous products.

The meaning of oligopoly is a market where several companies or sellers keep certain homogeneous products or differentiated products captive. In other words, an oligopoly is a state of the market where the industry has a few types of products or a few firms that dominate the market, or industry. This affects the parameters of the market, which includes costing. An individual firm can decide the cost of their products, and this decision affects the entire market. In an oligopoly, every individual firm has control over the market and its prices. To be up to the mark and ahead of other firms, individual firms have to compete with each other.

Types of Oligopoly

Oligopoly is of two kinds mainly: homogeneous oligopoly and heterogeneous (differential) oligopoly.

A homogeneous oligopoly is a firm or industry that sells or produces similar or homogeneous products. For instance, the steel industry has the same or similar products. Therefore, the steel industry can be classified as a homogeneous oligopoly.

A heterogeneous oligopoly is also known as a differentiated oligopoly. In a heterogeneous oligopoly, the firms produce nearly similar products as in a homogeneous oligopoly, but the products differ according to specific parameters. For example, the cosmetics industry produces similar products but they market these products in a way that displays its superiority over its competition. In this manner, companies try to gain a better standing in the market by introducing different features of their products.

Characteristic Features of an Oligopoly

These are the characteristic features of an oligopoly:

Number of firms

The primary characteristic of an oligopoly is that it has to have several firms that rule the market. If there is only one firm, it will become a monopoly, if there are two firms, it will become a duopoly, and if there are many companies, it will become a perfect competition. So in an oligopoly market, there should be a few firms or companies.

Interdependent firms

The second characteristic of an oligopoly is that the firms are interdependent. For instance, if a company changes the prices of its products, then the other companies will also have to change their prices to beat the competition. Similarly, companies have to know the special features of other companies' products to make similar products and market them. If a company fails to do this and moves ahead along with the market, it may undergo a loss.

Advertise

The third characteristic of an oligopoly is that companies have to advertise their products. In a monopoly, people know there is only one company for a certain type of product, so there is no advertising required. In a duopoly, advertising focuses on the different features of a company's products. In an oligopoly, in addition to advertising the different features, companies also have to advertise themselves so the general public knows who they are. The more a company advertises its products, the more it will be ahead of their competition.

Exit and entry of firms

The fourth characteristic of an oligopoly is the entry and exit of firms. Exiting from an oligopoly market is easy compared to entering an oligopoly market. There are several barriers that a new company might face while entering the market. Companies that are already a part of the market may prevent the entrance of other companies. This happens because as the number of companies increases in the market, the profit margin will decrease, and the competition will increase. These barriers can be in the form of government policies, licences, patents, high capital requirements, complex technology, and more.

Lack of uniformity

The fifth feature of an oligopoly is the lack of uniformity. One such example is that firms can be of different sizes. This means that in an oligopoly, it is not necessary for all firms to be the same size.

Competition

The sixth characteristic is competition. This is very evident in an oligopoly. Since the number of firms is less in an oligopoly, compared to perfect competition, the competition is intense.

Group behaviour

The seventh feature is group behaviour. This means that as a company changes its features or product prices, others will follow.

Conclusion

Oligopoly is a type of imperfect market competition. It is imperfect because the sizes of the companies are not the same. Moreover, any change in the way an individual company operates will change the market. This is called the group behaviour of companies. It is difficult for a new company to enter into an oligopoly market as either governments or firms issue certain restrictions.

A model of oligopoly was 1st put forward by Cournot a French economist in 1838. Cournot's model of oligopoly is one of the oldest theories of the behaviour of the individual firm and relate to non-collusive oligopoly. In the Cournot model it is assumed that an oligopolist thinks that his rival will keep their output fixed regardless of what he might do.

Another important model of non-collusive oligopoly was put forward by E.H. Chamberlin in his famous work "The theory of Monopolistic Competition". Chamberlin made an important improvement over the classical models of oligopoly, including that of Cournot. In sharp contrast to Cournot Chamberlin recognised in his model that oligopoly firms recognise their inter-dependence while fixing their output and price.

Cournot's Duopoly Model

Augustine Cournot, a French economist, published his theory of duopoly in 1838. But it remained mainly unnoticed till 1880 when Walras called the attention of the economists to Cournot's work.

Assumptions

- 1) Cournot takes the case of two identical mineral springs operated by two owners who are selling the mineral water in the same market. Their waters are identical. Therefore, his model relates to the duopoly with homogeneous products.
- 2) It is assumed by Cournot for the sake of simplicity that the owners operate mineral springs and sell water without incurring any cost of production.
- 3) The duopolists completely know the market demand for mineral water.
- 4) Cournot assumes that each duopolist believes that regardless of his actions and their effect on market price of the product, the rival firm will keep its output constant.

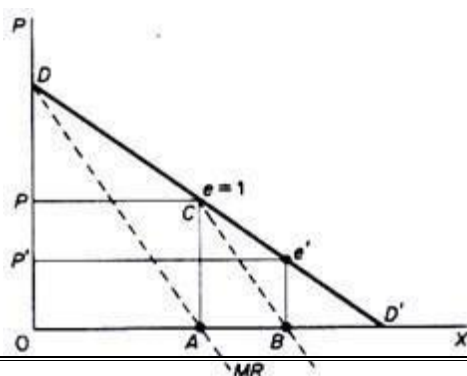


Figure 9.1

Assume that firm A is the first to start producing and selling mineral water. It will produce quantity A , at price P where profits are at a maximum because at this point $MC = MR = 0$. The elasticity of market demand at this level of output is equal to unity and the total revenue of the firm is a maximum. With zero costs, maximum R implies maximum profits, Π . Now firm B assumes that A will keep its output fixed (at $0/1$), and hence considers that its own demand curve is CD' .

Clearly firm B will produce half the quantity AD' , because (under the Cournot assumption of fixed output of the rival) at this level (AB) of output (and at price F) its revenue and profit is at a maximum. B produces half of the market which has not been supplied by A, that is, B's output is $\frac{1}{4}$ ($= \frac{1}{2} \cdot \frac{1}{2}$) of the total market.

Firm A, faced with this situation, assumes that B will retain his quantity constant in the next period. So he will produce one-half of the market which is not supplied by B. Since B covers one-quarter of the market, A will, in the next period, produce $\frac{1}{2}(1 - \frac{1}{4}) = \frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$ of the total market.

Firm B reacts on the Cournot assumption, and will produce one-half of the unsupplied section of the market, i.e. $\frac{1}{2}(1 - \frac{3}{8}) = \frac{5}{16}$.

In the third period firm A will continue to assume that B will not change its quantity, and thus will produce one-half of the remainder of the market, i.e. $\frac{1}{2}(1 - \frac{5}{16})$.

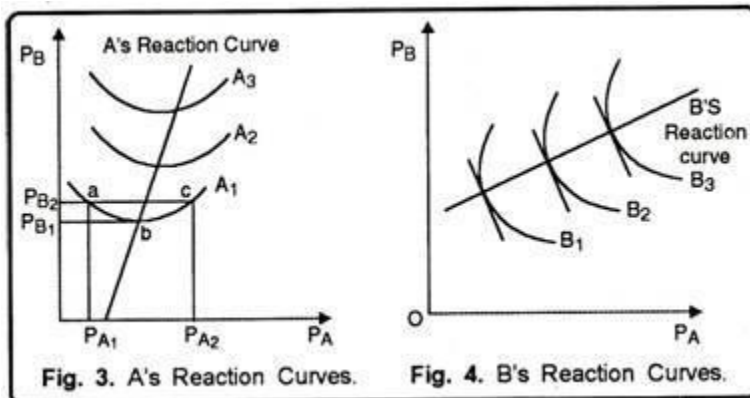
This action-reaction pattern continues, since firms have the naive behaviour of never learning from past patterns of reaction of their rival. However, eventually an equilibrium will be reached in which each firm produces one-third of the total market. Together they cover two-thirds of the total market. Each firm maximises its profit in each period, but the industry profits are not maximised.

Bertrand's Duopoly Model

Joseph Bertrand, French mathematician, criticised Cournot's duopoly solution and put forward a substitute model of duopoly. According to Bertrand there was no limit to the fall in price since each producer can always lower the price by undercutting the other and increasing the supply of output until the price becomes equal to the unit cost of product

Assumptions

- 1) The producers first set the price of the product and then produce the output which is demanded at that price.
- 2) Each producer believes that his rival will keep his price constant at the present level whatever price he himself set.
- 3) It is enough for the producer to know that he can capture the whole market by undercutting his rival.



Bertrand's model focuses on price competition. His analytical tools are reaction function of the duopolists. Reaction functions are derived on the basis of iso-profit curves. An iso-profit curve, for a give level of profit, is drawn on the basis of various combinations of prices charged by the rival firms. He assumed only two firms, A and B and their prices are measured along the horizontal and vertical axes, respectively.

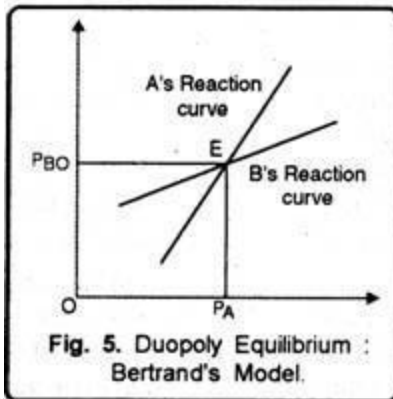
Their iso-profit curves are drawn on the basis of the prices of the two firms. Iso-profit curves of the two firms are concave to their respective prices axis, as shown in Fig. 3 and 4. Iso-profit curves of firm A are convex to its price axis P_A (Fig. 3) and those of firm B are convex to P_B (Fig. 4).

In Figure 4, we have curve A, which shows that A can earn a given profit from the various combinations of its own and its rival's price. For example, price combinations at points, a, b and c yield the same level of profit indicated by the iso-profit curve A_1 . If firm B fixes its price P_{B1} —firm A has two alternative prices, P_{A1} and P_{A2} , to make the same level of profits. When B reduces its price, A may either raise its price or reduce it. A will reduce its price when he is at point c and raise its price when he is at point a. But there is a limit to which this price adjustment is possible. This point is shown by point b. So there is a unique price for A to maximize its profits. This unique price lies at the lowest point of iso-profit curve.

The same analysis applies to all other iso-profit curves, A_1 , A_2 and A_3 we get A's reaction curve. Note that A's reaction curve has a rightward slant. This is so because, iso-profit curve tends to shift rightward when A gains market from his rival B.

Following the same process, B's reaction curve may be drawn as shown in Fig. 4.

The equilibrium of duopolists suggested by Bertrand's model may be obtained by putting together the reaction curves of the firms A and B as shown in Fig. 5.



The reaction curves of A and B intersect at point E where their expectations materialize, point E is therefore equilibrium point. This equilibrium is stable. For, if any one of the firms disagrees to this point, it will create a series of actions and reactions between the firms which will lead them back to point E.

Criticism of the Model:

Bertrand's model has been criticised on the same grounds as Cournot's model. Bertrand's implicit behavioural assumption that firms never learn from their past experience seems to be unrealistic. If cost is assumed to be zero, price will fluctuate between zero and the upper limit of the price, instead of stabilizing at a point.

TOPIC: Bertrand's Duopoly Model (With Diagram)

Bertrand developed his duopoly model in 1883. His model differs from Cournot's in that he assumes that each firm expects that the rival will keep its price constant, irrespective of its own decision about pricing.

Thus each firm is faced by the same market demand, and aims at the maximization of its own profit on the assumption that the price of the competitor will remain constant.

The model may be presented with the analytical tools of the reaction functions of the duopolists.

In Bertrand's model the reaction curves are derived from isoprofit maps which are convex to the axes, on which we now measure the prices of the duopolists. Each isoprofit curve for firm A shows the same level of profit which would accrue to A from various levels of prices charged by this firm and its rival.

The isoprofit curve for A is convex to its price axis (P_A). This shape shows the fact that firm A must lower its price up to a certain level (point e in figure 9.11) to meet the cutting of price of its competitor, in order to maintain the level of its profits at Π_{A2} . However, after that price level has been reached and if B continues to cut its price, firm A will be unable to retain its profits, even if it keeps its own price unchanged (at P_{Ae}). If, for example, firm B cuts its price at P_B , firm A will find itself at a lower isoprofit curve (Π_{A1}) which shows lower profits. The reduction of profits of A is due to the fall in price, and the increase in output beyond the optimal level of utilization of the plant with the consequent increase in costs. Clearly the lower the isoprofit curve, the lower the level of profits.

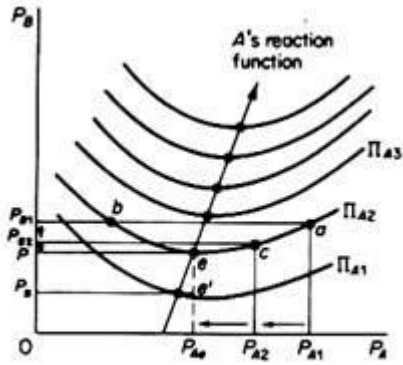


Figure 9.11

To summaries for any price charged by firm B there will be a unique price of firm A which maximizes the latter's profit. This unique profit-maximizing price is determined at the lowest point on the highest attainable isoprofit curve of A. The minimum points of the isoprofit curves lie to the right of each other, reflecting the fact that as firm A moves to a higher level of profit, it gains some of the customers of B when the latter increases its price, even if A also raises its price.

If we join the lowest points of the successive isoprofit curves we obtain the reaction curve (or conjectural variation) of firm A: this is the locus of points of maximum profits that A can attain by charging a certain price, given the price of its rival. The reaction curve of firm B may be derived in a similar way, by joining the lowest points of its isoprofit curves (figure 9.12).

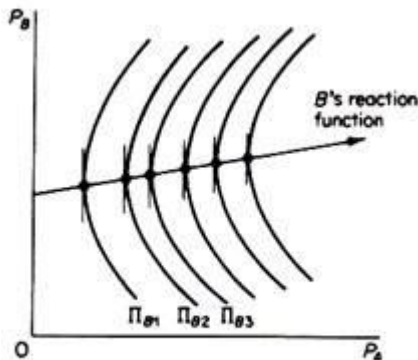


Figure 9.12

Bertrand's model leads to a stable equilibrium, defined by the point of intersection of the two reaction curves (figure 9.13). Point e denotes a stable equilibrium, since any departure from it sets in motion forces which will lead back to point e at which the price charged by A and B are P_{Ae} and P_{Be} respectively. For example, if firm A charges a lower price P_{A1} , firm B will charge P_{B1} , because on the Bertrand assumption, this price will maximize B's profit (given P_{A1}).

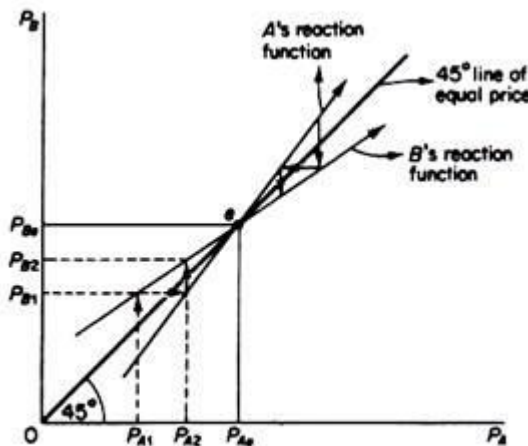


Figure 9.13

Firm A will react to this decision of its rival by charging a higher price P_{A2} . Firm B will react by increasing its price, and so on, until point e is reached, when the market will be in equilibrium. The same equilibrium will be reached if firms started by charging a price higher than P_{Ae} or P_{Be} a competitive price cut would take place which would drive both prices down to their equilibrium level P_{Ae} and P_{Be} .

Note that Bertrand's model does not lead to the maximization of the industry (joint) profit, due to the fact that firms behave naively, by always assuming that their rival will keep its price fixed, and they never learn from past experience which showed that the rival did not in fact keep its price constant. The industry profit could be increased if firms recognized their past mistakes and abandoned the Bertrand pattern of behaviour (figure 9.14).

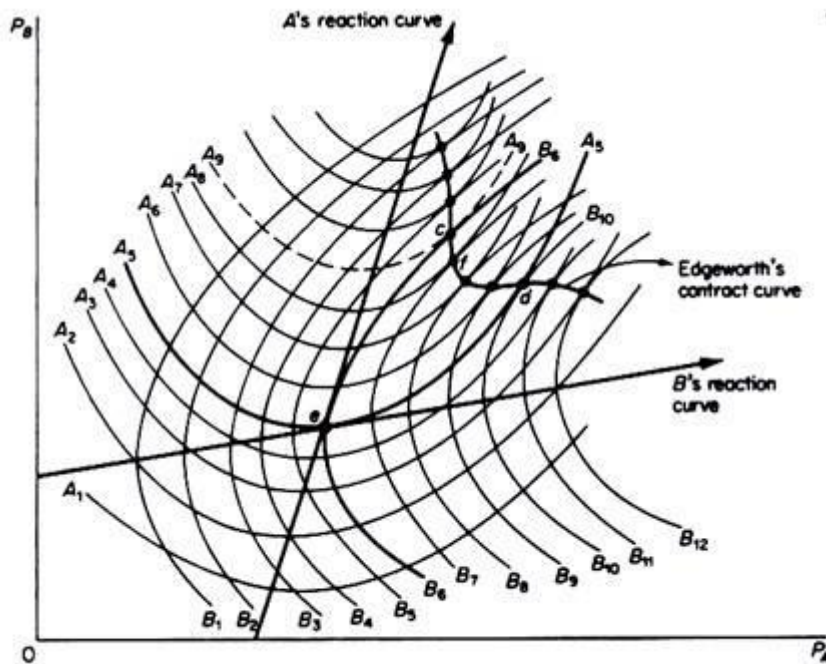


Figure 9.14

If firms moved on any point between c and d on the Edgeworth contract curve (which is the locus of points of tangency of the isoprofit curves of the competitors) one or both firms would have higher profits, and hence industry profits would be higher. At point c firm B would retain the same profit (B_6) as at point e, while A would move to a higher profit level (A_9). At point d firm A would have the same profit (A_5) as at the Bertrand equilibrium e, but firm B would move to a higher isoprofit curve (B_{10}). Finally, at any point between c and d (e.g. at f) both firms would realize higher profits (A_7 and B_5) as compared to those attained at Bertrand's solution ($A_7 > A_5$ and $B_5 > B_6$).

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Bertrand's model may be criticised on the same grounds as Cournot's model:

The behavioural pattern emerging from Bertrand's assumption is naive: firms never learn from past experience.

Each firm maximises its own profit, but the industry (joint) profits are not maximized.

The equilibrium price will be the competitive price. (In the example of costless mineral-water production, the price in Bertrand's model would fall to zero. If production is not costless, then price would fall to the level which would cover the costs of the duopolists inclusive of a normal profit.)

The model is 'closed'-does not allow entry.

The interesting feature of both Cournot's and Bertrand's models is that the limit of duopoly is pure competition. Neither model refutes the other. Each is consistent and is based on different behavioural assumptions. We may say that Bertrand's assumption (about the fixity of price of the rival) is more realistic, in view of the observed preoccupation of firms with keeping their

prices constant (except in cost inflation situations).

Furthermore, Bertrand's model focused attention on price setting as the main decision of the firm. The serious limitations of both models are the naive behavioural pattern of rivals; the failure to deal with entry; the failure to incorporate other variables in the model, such as advertising and other selling activities, location of the plant, and changes in the product.

Product differentiation and selling activities are the two main weapons of non-price competition, which is a main form of competition in the real business world; both models do not define the length of the adjustment process. Although dealing in terms of 'time periods,' their approach is basically static; both models assume that the market demand is known with accuracy; both models are based on individual demand curves which are located by making the convenient assumption of constant reaction curves of the competing firms.

Having discussed the classical duopoly models of Cournot and Bertrand, we proceed with the development of the traditional models of non-collusive oligopoly, which apply to market structures with a few firms conscious of their interdependence. It is worth while pointing out, however, that both Cournot's and Bertrand's models can be extended to markets in which the number of firms is greater than

PRICE LEADERSHIP AND BASING POINT PRICE LEADERSHIP MODELS

What Is Price Leadership?

Price leadership occurs when a leading firm in a given industry is able to exert enough influence in the sector that it can effectively determine the price of goods or services for the entire market. This type of firm is sometimes referred to as the price leader.

This phenomenon is common in industries that have oligopolistic market conditions, such as the airline industry. This level of influence often times leaves the rivals of the price leader with little choice but to follow its lead and match the prices if they are to hold onto their market share. In the airline industry, a dominant company typically sets the prices and other airlines feel compelled to adjust their prices to match the prices of the leading firm.

KEY TAKEAWAYS

- Price leadership occurs when a leading firm in a given industry is able to exert enough influence in the sector that it can effectively determine the price of goods or services for the entire market.
- There are three primary models of price leadership: barometric, collusive, and dominant.
- Price leadership is commonly used as a strategy among large corporations.
- There are certain economic conditions that make the emergence of price leadership more likely to occur within an industry, including a small number of companies in the industry, entry to the industry is restricted, products are homogeneous, and demand is inelastic.

How Price Leadership Works

There are certain economic conditions that make the emergence of price leadership more likely to occur within an industry: the number of companies involved is small; entry to the industry is restricted; products are homogeneous; demand is inelastic, or less elastic; organizations have a similar long-run average total cost (LRATC). LRATC is an economics metric that is used to determine the minimum (or lowest) average total cost at which a firm can produce any given level of output in the long run (when all inputs are variable).

The proliferation of price leadership tends to occur more often in sectors that produce goods and services that offer little differentiation from one producer to another.

Price leadership also tends to emerge when there is a high level of consumer demand for a

specific product; this results in consumers being drawn away from any competing products. Thus, the price of the specific product that is experiencing high levels of consumer demand becomes the market leader.

Types of Price Leadership

There are three primary models of price leadership: barometric, collusive, and dominant.

Barometric

The barometric price leadership model occurs when a particular firm is more adept than others at identifying shifts in applicable market forces, such as a change in production costs. This allows the firm to respond to market forces more efficiently. For instance, the firm may initiate a price change.

It is possible for a firm with a small market share to act as a barometric price leader if it's a good producer and if the firm is attuned to trends in its market. Other producers may follow its lead, assuming that the price leader is aware of something that they have yet to realize. However, because a barometric leader has very little power to impose its decisions on other firms in the industry, its leadership might be short-lived.

Collusive

The collusive price leadership model may emerge within markets that have oligopolistic conditions. Collusive price leadership occurs as a result of an explicit or implicit agreement among a handful of dominant firms to keep their prices in mutual alignment.

Smaller firms within the market are effectively forced into following the price change initiated by the dominant firms. This practice is most common in industries where the cost of entry is high, and the costs of production are known.

These agreements between firms—either explicit or implicit—may be considered illegal if the effort is designed to defraud the public. There is a fine line between price leadership and illegal acts of collusion. Price leadership is more likely to be considered collusive—and potentially illegal—if the changes in the price of a good are not related to changes in the operating costs of the firm.

Dominant

The dominant price leadership model occurs when one firm controls the vast majority of the market share in its industry. Within the industry, there are other, smaller firms that provide the same products or services as the leading firm. However, in this model, these smaller firms cannot influence prices.

A dominant price leadership model is sometimes referred to as a partial monopoly. In this type of model, the price leader might engage in predatory pricing, which refers to the practice of lowering prices to levels that make it impossible for smaller, competing firms to remain in business. In most countries, business decisions that enact predatory pricing and are aimed at hurting smaller companies are illegal.

Advantages and Disadvantages of Price Leadership

There are many potential advantages for firms that emerge as price leaders within an industry. In some instances, other firms within an industry may also benefit from the emergence of a price leader. For example, if companies in a particular market follow a price leader by setting higher prices, then all producers in that market stand to profit, as long as demand remains steady.

Price leadership also has the potential to eliminate (or reduce) price wars. If a market is completely comprised of companies of a similar size, in the absence of price leadership, price wars could ensue as each competitor tries to increase its share of the market.

One side effect of price leadership may be better-quality products as a result of an increase in profits. Increased profits often mean more revenue for companies to invest in research and development (R&D), and thus, an increase in their ability to design new products and deliver more value to customers.

The dynamics of price leadership may also create a system of interdependence rather than rivalry. When firms in the same market choose a parallel pricing structure—instead of undercutting each other—it fosters a positive environment conducive to growth for all companies.

There are also many potential disadvantages to the emergence of price leadership within an industry. In general, price leadership is only advantageous to businesses (in terms of their profits and performance). Price leadership where prices are increased does not convey any material advantages to consumers---however in the case where the price leader lowers prices consumers may benefit with less expensive goods and services.

In every price leadership model--barometric, collusive, dominant--it is the sellers that benefit from increased revenues, not the consumers. Customers will need to pay more for items that they were used to getting for less (before the sellers conspired to raise prices).

Consumers, however, may benefit in the short run if a price leader lowers prices. This assumes the price leader is not using predatory pricing to drive firms not able to respond out of business and later on exert monopoly pressure and raise prices.

Price leadership can also be unfair to smaller firms because small firms who attempt to match a leader's prices may not have the same economies of scale as the leaders. This can make it hard for them to sustain consistent price declines (and, in the long-term, to remain in business).

Price leadership can also result in malpractices on the part of competing firms that make the decision not to follow the leader's prices. Instead, they may engage in aggressive promotion strategies, such as rebates, money-back guarantees, free delivery services, and installment payment plans.

Finally, in a price leadership model, there is an inevitable discrepancy between the benefits conferred to the price leader versus the benefit conferred to other firms operating in the same industry. For example, if it costs the price leader less capital to produce the same product than it costs another firm, then the leader will set lower prices. This will result in a loss for any firm that has higher costs than the price leader.

Monopsony Pricing.

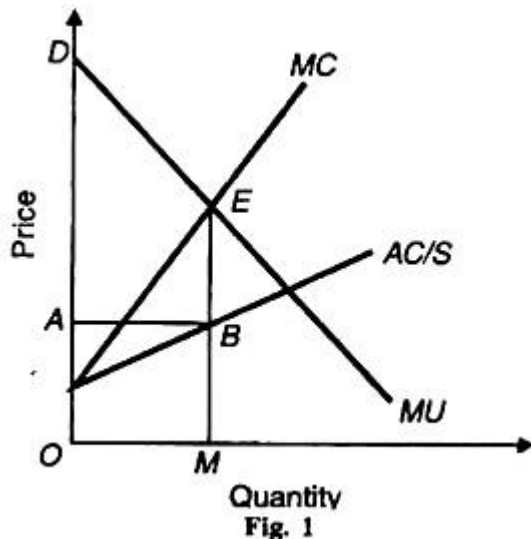
Monopsony refers to a market situation when there is a single buyer of a commodity or service. It applies to any situation in which there is a 'monopoly' element in buying.

For example, when consumers of a certain commodity are organised, or when a socialist government regulates imports, or when a certain individual happens to have a taste for some commodity which no one else requires, or when a single big factory in an isolated locality is the sole buyer of some grades of labour, there is monopsony.

Monopsony can be formally defined, in the words of Liebhafsky, as **“the case of a single buyer who is not in competition with any other buyers for the output which he seeks to purchase, and as a situation in which entry into the market by other buyers is impossible”**.

The analysis of monopsony pricing is similar to that under monopoly pricing. Just as a monopolist is able to influence the price of the product by the amount he offers for sale, similarly the monopolist is able to influence the supply price of his purchases by the amount he buys.

Again, the monopolist aims at the maximisation of his profits, while the monopolist aims at the maximisation of his surplus. The monopolist equates his marginal cost with his marginal D revenue to maximise his profits. The monopolist regulates his purchases in such a way that marginal cost equals marginal utility whereby his consumer's surplus is the maximum.



The determination of price under monopsony is explained in Figure 1. The supply curve of the industry is the average cost of the monopolist. It is from the industry that he buys the product. This is represented by the curve AC/S in the figure. MC is its corresponding marginal cost curve. MU is the marginal utility curve of the monopolist.

The monopolist equilibrium is established at E where the marginal utility of the product to the monopolist equals the marginal cost. He buys OM quantity at OA (=MB) price which is the supply price for that output. The surplus obtained by the Quantity monopolist is the area DEBA—the difference between what he is prepared to pay ODEM and what he actually pays OABM e.g., (DEBA = ODEM – OABM).

UNIT-II

TOPIC: The Theory Of Distribution

It is also known as the theory of factor pricing.

There are two aspects of this theory:-

1. Personal Distribution:- Income distribution among individuals is known as personal distribution. The theory of personal distribution studies how personal income of individuals are determined and what causes the inequalities in their income.
2. Functional Distribution:- This refers to how income is to be distributed among various factors of production. The theory of functional distribution studies how the relative prices of factors of production are determined. That is the reason why it has also been termed as the theory of factor prices.

Theory Of Distribution

- Theory of distribution is a specific matter of study of the theory of price. As the prices of products can be described with the help of the interaction of the demand and supply forces, likewise its distribution can also be described by the determination of prices of the factors which are also explained with interaction of their demand and supply forces
- But the important point to keep in mind is that In the theory of distribution, we depict the prices of the services rendered by the factors of production and not of the factors of production
- For example, in the market of the factors of production, it is not the labour which is being bought or sold, but the services of labour. Similarly, land, capital, and entrepreneur goods are not being evaluated, but the services of such land, capital, and entrepreneur

goods

- Thus, rent is not the price of land but the price of services or use of land & wages is the price of the service of Labour & interest is the price of the use of capital, and profit is the reward of entrepreneur's services
- Hence we can observe from the above mentioned points that the need for the theory of distribution (or the theory of factor pricing) arose due to the shortcomings in the theory of product pricing

Classical Theory of Distribution

According to the classical theory of distribution, the prices of the services of factors of production are determined by the supply and demand forces of such services.

- Rent:- As per the theory of rent propounded by Ricardo, rent is that portion of the produce of earth which is paid to the landlord for the original and indestructible powers of soil. He viewed rent as the differential surplus that some plots of land earn over and above the other
- Wages:- As per the theory of wages propounded by economist T. R. Mealthis and J. S. Mill, labour is sold in the market and its value is determined like the other commodities. According to J. S. Mill, the wage rate depends on the ratio of workforce to the amount of working capital
- Interest:- As per the theory of interest propounded by economist J. S. Mill, the interest is determined by the interaction of demand and supply forces of capital. The demand for capital is made for the purpose of investment and thus the demand for capital varies inversely with the rate of interest.
- Profit:- As per this theory of profit, the value of a commodity is determined on the basis of the services of labour used in it

Marginal Productivity Theory Of Distribution

Marginal productivity theory was given by German Economist Von Thunen in 1826. According to this theory, prices of the factors of production depend on its productivity and it is determined by the marginal productivity of the factor concerned. In other words, under perfect competition, every factor of production gets remuneration equal to its marginal productivity.

- Marginal Productivity:- It refers to the additional unit of product generated due to the employment of an additional unit of a factor of production.

Value of Marginal Physical Productivity = $MPP \times \text{Price}$

Avg. Gross Revenue Product = $TR / \text{No. of variables}$

Equilibrium where $MRP = \text{price of the factor}$

If $MRP > p$, increase the quantum of the factor concerned

If $MRP < p$, decrease the quantum of the factor concerned

- Assumptions:- Assumptions of this theory are as follows:
 1. Perfect market competition in product market
 2. Variable proportion type production function (i.e. output can be increased by changing the factor ratio)
 3. Divisible factor
 4. Full employment
 5. Constant technology
 6. Factors can be substituted
 7. One variable input and the other is fixed

Conclusion

This article throws light upon the theory of distribution. We have so far analysed that the theory of distribution has been viewed differently by the classical economists and the modern economists. Accordingly, many theories have been propounded by economists in this regard. We have so far covered the classical theory of distribution and the marginal productivity theory of distribution.

The Marginal Productivity Theory of Distribution (With Diagram)

The below mentioned article provides a close view on the marginal productivity theory of distribution.

Subject Matter:

The marginal productivity theory of distribution, as developed by J. B. Clark, at the end of the 19th century, provides a general explanation of how the price (of the earnings) of a factor of production is determined.

In other words, it suggests some broad principles regarding the distribution of the national income among the four factors of production.

According to this theory, the price (or the earnings) of a factor tends to equal the value of its marginal product. Thus, rent is equal to the value of the marginal product (VMP) of land; wages are equal to the VMP of labour and so on. The neo-classical economists have applied the same principle of profit maximisation ($MC = MR$) to determine the factor price. Just as an entrepreneur maximises his total profits by equating MC and MR, he also maximises profits by equating the marginal product of each factor with its marginal cost.

Assumptions of the Theory:

The marginal productivity theory of distribution is based on the following seven assumptions:

1. Perfect competition in both product and factor markets:

Firstly, the theory assumes the perfect competition in both product and factor markets. It means that both the price of the product and the price of the factor (say, labour) remains unchanged.

2. Operation of the law of diminishing returns:

Secondly, the theory assumes that the marginal product of a factor would diminish as additional units of the factor are employed while keeping other factors constant.

3. Homogeneity and divisibility of the factor:

Thirdly, all the units of a factor are assumed to be divisible and homogeneous. It means that a factor can be divided into small units and each unit of it will be of the same kind and of the same quality.

4. Operation of the law of substitution:

Fourthly, the theory assumes the possibility of the substitution of different factors. It means that the factors like labour, capital and others can be freely and easily substituted for one another. For example, land can be substituted by labour and labour by capital.

5. Profit maximisation:

Fifthly, the employer is assumed to employ the different factors in such a way and in such a proportion that he gets the maximum profits. This can be achieved by employing each factor up to that level at which the price of each is equal to the value of its marginal product.

6. Full employment of factors:

Sixthly, the theory assumes full employment for factors. Otherwise each factor cannot be paid in accordance with its marginal product. If some units of a particular factor remain unemployed, they would be then willing to accept the employment at a price less than the value of their marginal product.

7. Exhaustion of the total product:

Finally, the theory assumes that the payment to each factor according to its marginal productivity completely exhausts the total product, leaving neither a surplus nor a deficit at the end.

Some Key Concepts:

The theory is also based on key certain concepts.

These are the following:

1. MPP:

The first is marginal physical product of a factor. The marginal physical product (MPP) of a factor, say, of labour, is the increase in the total product of the firm as additional workers are employed by it.

2. VMP:

The second concept is value of marginal product. If we multiply the MPP of a factor by the price of the product, we would get the value of the marginal product (VMP) of that factor.

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3. MRP:

The third concept is marginal revenue product (MRP). Under perfect competition, the VMP of the factor is equal to its marginal revenue product (MRP), which is the addition to the total revenue when more and more units of a factor are added to the fixed amount of other factors, or $MRP = MPP \times MR$ under perfect competition. It is simply MPP multiplied by constant price, as $P = MR$. [VMP of a factor = MPP of the factor \times price of the product per unit, and MRP of a factor = MPP of the factor \times MR under perfect competition. So under perfect competition VMP of a factor = MRP of that factor.]

The Essence of the Theory:

The theory states that the firm employs each factor up to that number where its price is equal to its VMP. Thus, wages tend to be equal to the VMP of labour; interest is equal to VMP of capital and so on. By equating VMP of each factor with its cost a profit-seeking firm maximises its total profits. Let us illustrate the theory with reference to the determination of the price of labour, i.e., wages.

Let us suppose that the price of the product is Rs. 5 (constant) and the wages per unit of labour are Rs. 20 (constant). As the number of factors other than labour remain unchanged, wages represent the marginal cost (MC).

Table 12.1: Calculation of MPP, VMP and MRP of a Variable Factor (Labour)

Land	Capital	Labour	Total Product	MPP of Labour	VMP or MRP of Labour	The Wage Rate <i>AW = MW</i>
1 unit	1 unit	1 unit	10 units	×	×	Rs. 20
"	"	2 units	16 "	6 units	Rs. 30	"
"	"	3 units	21 "	5 units	Rs. 25	"
"	"	4 units	25 units	4 units	Rs. 20	"
"	"	5 units	28 "	3 units	Rs. 15	"
"	"	6 units	30 "	2 units	Rs. 10	"

Table 12.1 shows that at 2 or 3 labourers, the VMP or MRP of labour is greater than wages; so the firm can earn more profits by employing an additional labour. But at 5 or 6 labourers, the VMP or MRP of labour is less than wages, so it would reduce the number of labourers. But when it employs 4 labourers, the wage rate (Rs. 20) becomes equal to the VMP or MRP of labour (also Rs. 20). Here the firm gets the maximum profits because its marginal cost of labour (or marginal wage Rs. 20) is equal to its marginal revenue (VMP or MRP, Rs. 20).

Thus, under the assumption of perfect competition a firm employs a factor up to that number at which the price of the factor is just equal to the value of the marginal product (=MRP of the factor). In the same way it can be shown that rent is equal to the VMP of land, interest is equal to the VMP of capital, and so forth.

The theory may now be illustrated diagrammatically. See Fig. 12.2. Here WW is the wage line indicating the constant rate of wages at each level of employment ($AW = MW$. Here AW is average wage and MW is marginal wage). The VMP line shows the value of marginal product curve of labour, and it goes downwards from left to right indicating diminishing MPP of labour. Fig. 12.2 shows that the firm employs OL number of labourers, because by doing so it equates the MRP of labour with the wage ratio, and makes optimum purchase of labour.

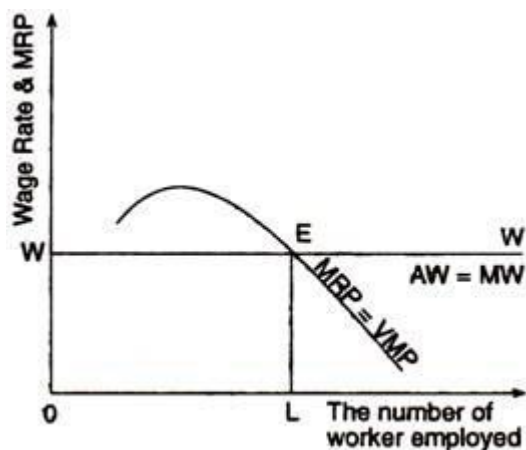


Fig. 12.2. Wage Determination

Criticisms of the Theory:

The marginal productivity theory of distribution has been subjected to a number of criticisms:

1. In determination of marginal product:

Firstly, main product is a joint product— produced by all the factors jointly. Hence the marginal product of any particular factor (say, land or labour) cannot be separately determined. As William Petty pointed out as early in 1662: Labour is the father and active principle of wealth, as lands are the mother.

2. Unrealistic:

It is also shown that the employment of one additional unit of a factor may cause an improvement in the whole of organisation in which case the MPP of the variable factors may increase. In such circumstances, if the factor is paid in accordance with the VMP, the total product will get exhausted before the distribution is completed. This is absurd. We cannot think of such a situation in reality.

3. Market imperfection:

The theory assumes the existence of perfect competition, which is rarely found in the real world. But E. Chamberlin has shown that the theory can also be applied in the case of monopoly and imperfect competition, where the marginal price of a factor would be equal to its MRP (not to its VMP).

4. Full employment:

Again, the assumption of full employment is also unrealistic. Full employment is also a myth, not a reflection of reality.

5. Difficulties of factor substitution:

W. W. Leontief, the Nobel economist, denies the possibility of free substitution of the factors always owing to the technical conditions of production. In some products process, one factor cannot be substituted by another. Moreover organisation or entrepreneurship is a specific factor which cannot be substituted by any other factor.

6. Emphasis on the demand side only:

The theory is one-sided as it ignores the supply side of a factor; it has emphasised only the demand side i.e., the employer's side, in the opinion of Samuelson, the marginal productivity theory is simply a theory of one aspect of the demand for productive services by the firm.

7. Inhuman theory:

Finally, the theory is often described as 'inhuman' as it treats human and non-human factors in the same way for the determination of factor prices.

UNIT-III

TOPIC: Pigovian Welfare Economics (With Evaluation)

Arthur Cecil Pigou succeeded Prof. Marshall as the Professor of Economics at the University of Cambridge. After Marshall, he became the leading neo classical economist. He is the founder of “Welfare Economics” His leading ideas on welfare economics are found in his “Economics of Welfare” (1920). Prof. Pigou popularized the word welfare and gave a concrete meaning to it.

In his book, Pigou dealt with three things:

- (1) A definition of economic welfare
- (2) Spelling out the condition under which welfare is maximised and
- (3) Pronouncement of policy recommendations for increasing welfare.

Prof. Pigou gave a clear meaning to the concept of welfare. He defined individual welfare as the sum of satisfactions obtained from the use of goods and services. Social welfare is the summation of all individual welfare in a society. Since general welfare is very wide and complicated, he limited his study to economic welfare. He defined economic welfare as that part of social welfare “that can be brought directly or indirectly into relation with the measuring rod of money.”

Pigou regards economic welfare and national income as coordinate.

He lays down two conditions for maximizations of welfare:

- (i) Given the taste and income distribution, an increase in national income represents an increase in welfare,
- (ii) For welfare maximisation, the distribution of national income is equally important.

If national income remains constant, transfer of income from rich to the poor would improve welfare. With income subject to diminishing marginal utility, transfers of income from the rich to the poor will increase social welfare by satisfying the more intense wants of the poor. Thus it is economic equality that maximises welfare.

Prof. Pigou had a dual criterion for detecting the increase in social welfare. First, he measured the economic welfare of the society in money value and thus, given the supply of resources, an increase in national dividend meant an increase in social welfare.

Second, Pigou favoured an income equalisation policy and therefore, reorganization of the economy which increases the share of the poor without offsetting adversely “productive effort enterprise and development of capital equipment was to be taken as a gain in social welfare.”

Pigou has made a distinction between private and social costs. The private marginal cost of a commodity is the cost of producing an additional unit. The social marginal cost is the expense or damage to society as a consequence of producing that commodity. Private marginal benefit can be measured by the selling price of the commodity.

Social marginal benefit refers to the total benefit that society gets from the production of an additional unit. By making a distinction between social and private valuations of economic activity, he paved the way for the analysis of external effects or externalities in social welfare economics.

The presence of external effects in production was seen by Prof. Pigou in the divergence between social net product and private net product. He defined social net product “as the aggregate contribution made to the national dividend” and the private net product as the contribution which is capable of being sold and the proceeds added to the earnings of the person responsible for investment.

The divergence between the two products shows itself in the form of external effects of production associated with marginal increments of output. In some cases social net product is more than the private product while in others private product is greater than the social product. As an example of the former, Pigou pointed out to the greater social benefit from technical training of workers by a private firm.

As an illustration of the latter he cited the fact that the smoke rising from the chimneys of private

factory spoils the atmosphere of the locality and increases the laundry bills of the people of the neighbourhood. But people are not compensated in any way by the factory owner.

He was of the opinion that the state should equalize the private net product with social net product, if in an industry where private net product is more, it should be taxed, and if another industry shows a lesser private net product, it ought to be subsidized. Of course, Prof. Pigou recognised that the divergence between private net product and social net product cannot always be quantified and measured in terms of money.

Prof. Pigou made the first attempt to lay down the conditions of social optimum which he termed "the ideal output" of the economic system as a whole. In his view, the social optimum prevails when marginal social products are equal in all industries and thus production of real wealth is maximised.

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Assuming that all the productive resources are being employed and that there is no cost of movement between different occupations and places, it can be concluded that the national dividend is the largest when the values of the marginal social net products are equal in all industries. If this arrangement prevails, the society is having its "ideal output".

Pigovian welfare conditions presuppose the existence of the following assumptions:

1. It was assumed that everybody as consumer acts rationally to maximise his satisfaction.
2. Pigou believed that utility is not measurable cardinally. But inter personal and intra personal comparisons of utilities are also possible so that it is possible to find out quantitatively whether welfare has increased or decreased.
3. Pigou put forward a basic postulate that a man possesses equal capacity for satisfaction when placed in similar circumstances. Pigou held that different people derive the same satisfaction out of the same real income.
4. Pigou also believed that the marginal utility of money income falls as money income increases. As a result, the marginal utility of an addition to the income of a poor man is greater as compared to the loss of utility from the loss of the same amount of income to a rich man.

Evaluation:

Pigou provided the first systematic theoretical base of welfare economics and integrated the normative problems with the positive ones. He provided a rationale for state intervention at places where private and social net product diverged. But his policy recommendations were all value based. As such his study was more normative than theoretical.

Though Pigou's Economics of Welfare was the first clear analysis of welfare economics, yet the Pigovian conditions of welfare have been criticised on the following grounds. Pigou lays emphasis on the maximisation of welfare, but he does not clarify the notion of maximisation.

Pigovian assumption of equal capacity for satisfaction is scientifically untenable. This represents a broad value judgement in favour of equal distribution of wealth. The capacity for satisfaction of any individual is a subjective thing incapable of objective quantification.

Another trouble with Pigovian welfare economics, is the lack of rigour and operational content in the distinction between private and social products. Pigou seems to have assumed that the divergence between the two is not inherent in the working of the free enterprise system. It is traceable to and can be corrected through governmental intervention. In the real world, structural failures resulting from immobility, indivisibility and imperfect knowledge are so numerous as to defy correction through social action.

The classification of general welfare into 'economic' and 'non-economic' welfare has also been criticised as too superficial to be made the basis of all welfare analysis. The most destructive criticism of the Pigovian welfare economics was the unrealistic nature of the assumptions of cardinal additivity of the individual utility functions to get the social welfare functions. Economists do not agree with this view because quantitative measurement of utility is not possible.

Pigou's welfare conditions are related to national income. But it is not easy to calculate national income. Again, social welfare does not increase by a mere increase in national income. It is

possible that national income may increase due to inflationary rise in prices and poor may become worse off than before.

Welfare economics is closely related to ethics but Pigou does not clarify it. Welfare economics is essentially a normative study in which value judgements and inter personal comparisons are made. By not relating these concepts with his notion of welfare, Pigou's economics of welfare is not considered as an objective study of the causes of welfare.

Marshall made the concept of consumers surplus as the central tool of his welfare analysis. Marshall considered the community's welfare to be maximum when its satisfaction is maximum.

There were two basic assumptions of this maximum welfare analysis of Dr. Marshall:

1. Equal sums of money measure equal utilities to all and
2. A fall in the price of the product results in a fall in output and hence loss of satisfaction.

Marshall advocated that an industry which is working under increasing returns must produce beyond its equilibrium point and an industry working with diminishing returns should stop producing before their equilibrium output. For this he suggested the policy of giving a bounty to increasing returns industry and levying a tax on the industries subject to diminishing returns. The proceeds of the tax could be used for giving bounties.

Topic:Pareto Optimality: Conditions and Composition

Contents:

1. Introduction to Pareto Optimality
2. Efficiency in Production
3. Pareto Optimality in Production and Perfect Competition
4. Efficiency in Consumption or Exchange
5. Pareto Optimality in Consumption or Exchange and Perfect Competition
6. Pareto Optimality Conditions when the External Effects are Present
7. Efficiency in the Allocation of Factors among Commodities, or, Efficiency in Product-Mix or Composition of Output
8. Pareto-Optimal Composition of Outputs and Perfect Competition

1. Introduction to Pareto Optimality:

The welfare of a society depends, in the broadest sense, upon the satisfaction levels of all its consumers. But almost every change in the economic state of the society will have favourable effects on some members and unfavorable effects on others.

Evaluation of such a social change is impossible unless the economist is ready to go into interpersonal comparison of utility under some value judgement, which he may not be willing to do. Rather, he will be willing to evaluate such changes where at least one person has been better off and no one worse off.

The Italian economist Vilfredo Pareto (1848-1923) said that if a change in the economic state makes at least one individual better off without making anyone worse off, then the change is for the betterment of social welfare, i.e., the change is desirable. In that case, we say that the initial state was Pareto-non-optimal.

On the other hand, if a change makes no one better off and at least one worse off, implying that the change will make the society worse off, then, from the point of view of welfare, the initial economic state is Pareto-optimal.

Therefore, the Pareto optimality criterion can be stated in this way:

A situation in which it is impossible to make any one better off without making someone worse off, is said to be Pareto optimal or Pareto-efficient.

Obviously, the concept of Pareto optimality avoids interpersonal comparison of utility. Since most government policies involve changes in the economic state, which benefit some people and bring discomforts to others, it is obvious that the concept of Pareto optimality is of limited

applicability in the real world situations.

Pareto Optimality Conditions:

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For the attainment of Pareto-efficient situation in an economy, three marginal conditions must be satisfied.

These are:

- (i) Marginal condition for efficiency in the allocation of factors among firms (efficiency in production);
- (ii) Marginal condition for efficiency of distribution of commodities among consumers (efficiency in consumption); and
- (iii) Marginal condition for efficiency in the allocation of factors among commodities (efficiency in product-mix or composition of output).

Assumption:

In order to derive these three marginal conditions for the attainment of Pareto optimality, we shall assume, for the sake of simplicity, that there are only two consumers (I and II), two factors of production (X_1 and X_2), and two commodities (Q_1 and Q_2), i.e., our model here would be a $2 \times 2 \times 2$ model.

2. Efficiency in Production:

If we assume that the consumer goods are of “**more is better**” type and that external effects are absent in consumption, then an increment in the quantity produced of at least any one consumer good without a decrement in the quantity of any other, can lead to an improvement in utility level of at least one consumer without utility decrements for others.

Therefore, Pareto optimality in production requires that the output level of each consumer good be at a maximum, given the output levels of all other consumer goods.

We may derive the marginal condition for Pareto-efficiency in production with the help of Fig. 21.1 which is called an Edgeworth box diagram. The dimensions of the rectangle in Fig. 21.1 represent the total available quantities, and x^0_2 , of the inputs X_1 and X_2 that would all be used to produce the consumer goods Q_1 and Q_2 .

Any point in the box represents a particular allocation of the inputs over the production of the two goods.

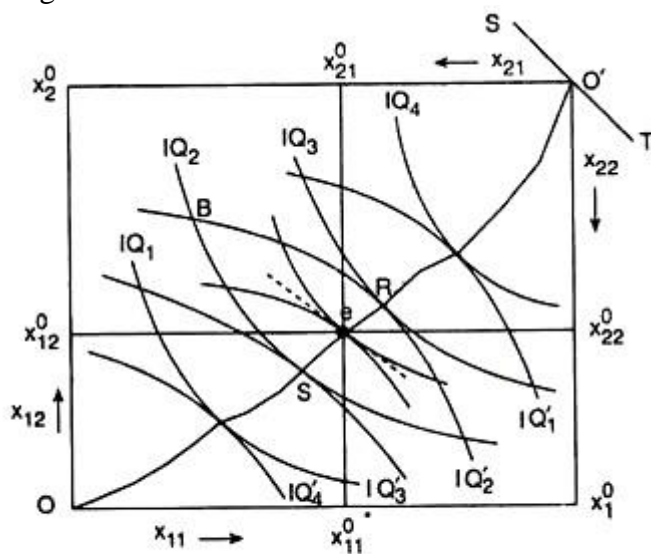


Fig. 21.1 Edgeworth contract curve for production

For example, if the allocation of the inputs is given by the point B, the quantities of X_1 and X_2 used in the production of good Q_1 are measured by the coordinates of B with reference to the origin O, and the quantities of X_1 and X_2 used in the production of good Q_2 are measured by the coordinates of point B with reference to the origin O'.

The isoquant (IQ) maps for goods Q_1 and Q_2 are given in Fig. 21.1 with reference to the points of origin O and O' , respectively.

Now, the marginal condition for Pareto efficiency in production would be obtained if we maximise the output of good Q_1 subject to a given output level of good Q_2 . Such maximisation would occur at a point of tangency between the IQs for the two goods.

For example, maximisation of output of Q_1 subject to the quantity of Q_2 as given by IQ_3 , would occur at the point of tangency S between the IQs for the goods. Similarly, maximisation of output of Q_2 subject to the quantity of Q_1 as given by IQ_3 , would occur at the point of tangency R between the IQs for the two goods.

However, at the point of tangency between the IQs for the two goods, we have numerical slope of IQ for good Q_1 = numerical slope of IQ for good Q_2

$MRTS_{X_1, X_2}$ in the production of Q_1 = $MRTS_{X_1, X_2}$ in the production of Q_2 (21.1)

Thus, the marginal condition for Pareto efficiency in production is given by (21.1) which states that the marginal rate of technical substitution (MRTS) between the two inputs should be the same in the production of the two goods.

It is obvious from above that the Pareto efficiency point in production must necessarily be a point of tangency between the IQs for the two goods. If we join all the points of tangency between the IQs for the two goods, by a curve, we would obtain what is called the Edge-worth contract curve for production which we would denote by CCP. The CCP would run from the point O to the point O' in Fig. 21.1.

We have obtained then that all the points on the CCP are Pareto-efficient points in production. That is, if we are at some point on the CCP, then we are no longer able to effect by a change in the allocation of the inputs, an increase in the output of one of the goods without reducing the quantity of the other.

On the other hand, any point like B in Fig. 21.1, which does not lie on the CCP and which does not satisfy condition (21.1), is Pareto-non-optimal. At the point B , we are on IQ_2 for good Q_1 and on IQ'_2 for good Q_2 .

However, after a reallocation of the resources, if the economy reaches at some point on the CCP between R and S , then the quantities of both the goods would be larger, and if the economy reaches just at the point R or S , then the quantity of one of the goods would be larger and that of the other good would remain the same.

This shows that any point B that does not lie on the CCP, is Pareto-non-optimal, and, by a reallocation of the resources, if the economy is brought on to some point on the segment RS of the CCP, then at least one of the goods would be produced in a larger quantity, that of the other remaining the same.

We have seen that all the points on the CCP are Pareto-optimal. However, we cannot compare any two points, e.g., R and S , on the CCP because if the economy moves from S to R , the output of Q_1 would increase and that of Q_2 would decrease resulting in advantage for some people and disadvantage for some others, and since interpersonal comparison of utility is ruled out, we cannot compare the points R and S .

Mathematical Derivation of the Conditions:

We may also derive mathematically the marginal condition for Pareto efficiency in production.

Let us suppose that the production functions for the goods Q_1 and Q_2 are:

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$$q_1 = q_1(x_{11}, x_{12})$$

$$\text{and } q_2 = q_2(x_{21}, x_{22}) \quad (2.12)$$

where q_1 and q_2 are the quantities produced of goods Q_1 and Q_2 , x_{11} and x_{12} are the quantities of inputs X_1 and X_2 used in the production of Q_1 , and x_{21} and x_{22} are the quantities of these inputs used in the production of good Q_2 .

Since the total available quantities of the two inputs are x^0_1 and x^0_2 , we may write:

$$\begin{aligned} x_{11} + x_{21} &= x_1^0 \\ x_{12} + x_{22} &= x_2^0 \end{aligned} \quad (21.3)$$

As per the requirements of Pareto optimality, the efficiency conditions may be derived if we maximise q_1 as given by (21.2) subject to:

$$\begin{aligned} q_2 &= q_2^0 \\ \text{or, } q_2 - q_2^0 &= 0 \end{aligned} \quad (21.4)$$

where q_2^0 is any given quantity of good Q_2 .

The relevant Lagrange function for this constrained maximisation problem is:

$$\begin{aligned} Z &= q_1(x_{11}, x_{12}) + \mu [q_2(x_{21}, x_{22}) - q_2^0] \\ &= q_1(x_{11}, x_{12}) + \mu [q_2(x_1^0 - x_{11}, x_2^0 - x_{12}) - q_2^0] \end{aligned} \quad (21.5)$$

The first order or the necessary conditions for maximum q_1 subject to $q_2 = q_2^0$ are:

$$\left. \begin{aligned} \frac{\partial Z}{\partial x_{11}} &\equiv \frac{\partial q_1}{\partial x_{11}} - \mu \frac{\partial q_2}{\partial x_{21}} = 0 \\ \frac{\partial Z}{\partial x_{12}} &\equiv \frac{\partial q_1}{\partial x_{12}} - \mu \frac{\partial q_2}{\partial x_{22}} = 0 \\ \frac{\partial Z}{\partial \mu} &\equiv q_2(x_1^0 - x_{11}, x_2^0 - x_{12}) - q_2^0 = 0 \end{aligned} \right\} \quad (21.6)$$

From (21.6), we have

$$\begin{aligned} \frac{\frac{\partial q_1}{\partial x_{11}}}{\frac{\partial q_1}{\partial x_{12}}} &= \frac{\frac{\partial q_2}{\partial x_{21}}}{\frac{\partial q_2}{\partial x_{22}}} \\ \Rightarrow \text{MRTS}_{x_1, x_2} &= \text{MRTS}_{x_1, x_2} \end{aligned} \quad (21.7)$$

in the production of Q_1 in the production of Q_2

which is the same as condition (21.1).

Pareto efficiency condition (21.1) or (21.7) gives us that the available quantities of the two inputs, X_1 and X_2 , should be allocated over the production of the two goods, Q_1 and Q_2 , in such a way that the MRTS between the inputs may be the same in the production of the two goods.

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We may now see with the help of a simple example why condition (21.7) is necessary for Pareto efficiency in production. Let us suppose that in the production of Q_1 , $\text{MRTS}_{x_1, x_2} = 2$ and, in the production of Q_2 , $\text{MRTS}_{x_1, x_2} = 1$

i.e., the MRTS is not the same in the production of the two goods.

It follows from above that we can substitute 1 unit of X_1 for 2 units of X_2 in the production of Q_1 , and keep the output of Q_1 constant. Similarly, we can substitute 1 unit of X_1 for 1 unit of X_2 in the production of Q_2 , and keep the output of Q_2 constant. So, all we have to do is to take 1 unit of X_1 out of the production of Q_2 and use it in the production of Q_1 .

This releases 2 units of X_2 from the production of Q_1 , 1 unit of which may be transferred to the production of Q_2 to keep its output at the initial level. If we do all this, the output of Q_1 and Q_2 would remain unchanged, and yet we are left with an extra unit of X_2 . We can use this unit in the production of Q_1 (or Q_2) and get more of Q_1 (or of Q_2). Thus, one output is increased without reducing the other output.

The above example shows that if the MRTS_{x_1, x_2} in the production of the two goods are not

equal, if MRTS in the production of Q_2 is lower, say, than that in the production of Q_1 ; then we have to take away the marginal unit of input X_1 from the production of Q_2 and transfer it to the production of Q_1 where the $MRTS_{X_1, X_2}$ is higher, and take away from the field the input X_2 , in exchange.

As we continue the process, the MRTS in the production of Q_2 would rise as the quantity of X_1 falls, and the MRTS in the production of Q_1 would fall as the quantity of X_1 increases, and, as we have seen, the allocation becomes better in the Pareto sense.

Therefore, if we are to reach the Pareto-efficient situation, we have to continue the process till the MRTS becomes equal in the production of the two goods. For when the MRTS in the production of both the goods becomes the same, no further reallocation will be able to increase the production of at least one of the goods without reducing the production of the other good.

To understand this, let us suppose that the MRTS between the two inputs are equal in the production of the two goods, and it is equal to 4. In that case, if we take away 1 unit of X_1 from the production of Q_2 , and transfer it to the production of Q_1 , the latter would release 4 units of X_2 in exchange, so that the output level of Q_1 might remain constant.

These 4 units of X_2 should be transferred to the production of Q_2 because there the MRTS is 4, and when 4 units of X_2 are given to be used in the production of Q_2 in exchange for 1 unit of X_1 , the output of Q_2 would remain unchanged at the initial level.

Therefore, by means of a reallocation of the resources, we have not been able to increase the production of at least one of the goods. On the contrary, a reallocation of the inputs would keep the outputs of the two goods unchanged at their initial quantities.

3. Pareto Optimality in Production and Perfect Competition:

Pareto optimality in production is guaranteed under perfect competition. For, under perfect competition, the prices r_1 and r_2 of the two inputs, X_1 and X_2 , are given to the firms that produce the goods Q_1 and Q_2 , and each profit-maximising firm equates the $MRTS_{X_1, X_2}$ to the ratio of the prices of the inputs.

That is, for the producer of Q_1 we get:

$$\frac{\frac{\partial q_1}{\partial x_{11}}}{\frac{\partial q_1}{\partial x_{12}}} = \frac{r_1}{r_2}$$

and for the producer of good Q_2 , we get

$$\frac{\frac{\partial q_2}{\partial x_{21}}}{\frac{\partial q_2}{\partial x_{22}}} = \frac{r_1}{r_2}$$

(21.8)

From (21.8), we obtain:

$$MRTS_{X_1, X_2} \text{ in the production of } Q_1 = MRTS_{X_1, X_2} \text{ in the production of } Q_2 \quad (21.9)$$

Since condition (21.9) is the same as condition (21.7), Pareto efficiency in production is a certainty under perfect competition.

We may now obtain a graphical solution of equation (21.7) or (21.9) for the allocation of inputs X_1 and X_2 over the production of goods Q_1 and Q_2 and for the quantities produced of Q_1 and Q_2 . The satisfaction of the marginal condition (21.7) or (21.9) is guaranteed under perfect competition.

Let us suppose that in the competitive markets the prices of the inputs are given to be r_1 and r_2 .

Let us now draw a straight line ST of slope $-r_1/r_2$ through the point O' in Fig. 21.1, and pick up the point e on the contract curve for production (CCP) where the common slope of the isoquants has been equal to the slope of the line ST. That is, at the point e, we have numerical slopes of the IQs of two individuals = the numerical slope of the line ST = r_1/r_2

$$\Rightarrow MRTS_{x_1, x_2}^{Q_1} = MRTS_{x_1, x_2}^{Q_2} = \frac{r_1}{r_2} \quad (21.10)$$

That is, at the point e in Fig. 21.1, the marginal condition for efficiency of production has been satisfied. At this point quantities of the two inputs, x_{11}^0 and x_{21}^0 would be used in the production of Q_1 and these quantities, when substituted in the production function for Q_1 , would give us the output quantity of Q_1 . Similarly, quantities of the two inputs, x_{21}^0 and x_{22}^0 , would be used in the production of Q_2 and the output here would be q_2^0 .

4. Efficiency in Consumption or Exchange:

A distribution of the given quantities of the two commodities Q_1 and Q_2 among two consumers I and II is said to be Pareto-efficient if it is impossible, by a redistribution of these goods, to increase the utility of one individual without reducing the utility of the other.

The marginal condition for efficiency in consumption or exchange can be derived with the help of the Edgeworth box diagram given in Fig. 21.2. The dimensions of the rectangle in Fig. 21.2 represent the total available quantities, q_1^0 and q_2^0 , of the two goods in a pure-exchange economy.

Any point in the box represents a particular distribution of the commodities between the two consumers. For example, if the distribution of commodities is given by point A, the quantities of Q_1 and Q_2 consumed by consumer I are measured by the coordinates of A with respect to the origin O and the quantities of the two goods consumed by II are measured by the coordinates of A w.r.t. the origin O'.

The indifference map of consumer I has been given w.r.t. the origin O and that of II has been given w.r.t. the origin O'.

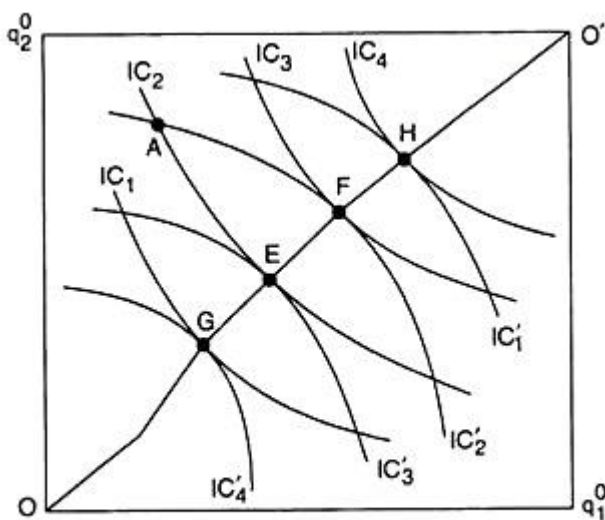


Fig. 21.2 Efficiency in consumption or exchange

Now, the marginal condition for Pareto efficiency in consumption or exchange would be obtained if we maximise the utility level of consumer I or II subject to the given utility level of consumer II or I. Such maximisation would occur at a point of tangency between the indifference curves (ICs) of the two consumers. For example, maximisation of utility of consumer I subject to the utility level of II as given by IC_1 of consumer II, would occur at the point of tangency, E, between the ICs of two consumers.

Similarly, maximisation of utility of consumer II subject to the utility level of I as given by IC_3 of consumer I would occur at the point of tangency, F, between the ICs of the two consumers. It may be added, therefore, that the exchange equilibrium is not unique.

Now, at the point of tangency between the ICs of the two consumers, we have numerical slope of IC of consumer I = numerical slope of IC of consumer II

$$\Rightarrow \text{MRS}_{Q_1, Q_2} \text{ of consumer I} = \text{MRS}_{Q_1, Q_2} \text{ of consumer II} \quad (21.11)$$

Thus, the marginal condition for Pareto efficiency in consumption is given by (21.11). It is obvious from above that any point of tangency between the ICs of two consumers is a Pareto efficiency point. If we join all such points of tangency by a curve in Fig. 21.2, we obtain what is known as the Edgeworth contract curve for consumption or Exchange (CCC or CCE), which would run from the point O to the point O'.

Therefore, all the points on the contract curve at which (21.11) is satisfied, are Pareto-efficient points in consumption. For, if we are at some point on the contract curve, in Fig. (21.2), we are not able to effect, by a change in the distribution of the goods, an improvement in the utility of one consumer without reducing the utility of the other.

Therefore, let us note again that the point of Pareto efficiency in exchange is not unique. On the other hand, any point like A, which does not lie on the contract curve and which does not satisfy (21.11), is Pareto-non-optimal. At the point, A, consumer I is on his IC₁ and consumer II is on his IC₂.

However, after a redistribution of the commodities, if the consumers are brought at some point on the contract curve between E and F, then both the consumers would benefit for both of them would reach now higher ICs, and if they are brought just at the point E or F, then one of them will benefit, while the utility level of the other will remain the same.

This shows that any point A, which does not lie on the CCE, is Pareto-non-optimal and by a redistribution of the commodities, if we bring the consumers on to the EF segment of the CCE, then at least one of them would benefit, the utility level of the other remaining the same.

We have seen that all points on the contract curve are Pareto-efficient. However, we cannot compare the points on the contract curve because that will involve interpersonal comparison of utility, which is not possible without an explicit value judgement.

Mathematical Derivation of the Conditions:

We may also derive mathematically the marginal condition for Pareto efficiency in consumption, or, Exchange. Let us suppose that the utility functions of the two consumers I and II are respectively,

$$u_1 = u_1(q_{21}, q_{12})$$

$$\text{and } u_2 = u_2(q_{21}, q_{22}) \quad (21.12)$$

where q_{11} and q_{12} are the quantities of Q_1 and Q_2 consumed by consumer I and q_{21} and q_{22} are the quantities of the two goods consumed by individual II.

If and q_2 are the given quantities of the two goods, then we have:

$$q_{11} + q_{21} = q_1^0$$

$$\text{and } q_{12} + q_{22} = q_2^0 \quad (21.13)$$

It is evident from (21.12) that the utility level of each consumer depends only upon the quantities consumed by him and not upon the quantities consumed by the other. That is, it has been assumed here that external effects are absent.

Pareto-efficiency in consumption implies that u_1 is maximised subject to a given $u_2 = u_2^0$, or, the other way round. Let us then form the relevant Lagrange function, V, for the constrained maximisation of u_1 as

$$V = u_1(q_{11}, q_{12}) + \lambda [u_2(q_{21}, q_{22}) - u_2^0] \quad (21.14)$$

where λ is the Lagrange multiplier.

Now, the first-order conditions for the constrained maximisation of u_1 subject to $u_2 = u_2^0$ are:

$$\left. \begin{aligned} \frac{\partial V}{\partial q_{11}} &\equiv \frac{\partial u_1}{\partial q_{11}} - \lambda \cdot \frac{\partial u_2}{\partial q_{21}} = 0 \quad [\because q_{21} = q_1^0 - q_{11}] \\ \frac{\partial V}{\partial q_{12}} &\equiv \frac{\partial u_1}{\partial q_{12}} - \lambda \frac{\partial u_2}{\partial q_{22}} = 0 \quad [\because q_{22} = q_2^0 - q_{12}] \\ \frac{\partial V}{\partial \lambda} &\equiv u_2(q_1^0 - q_{11}, q_2^0 - q_{12}) - u_2^0 = 0 \end{aligned} \right\} \quad (21.15)$$

From (21.15), we have:

$$\frac{\frac{\partial u_1}{\partial q_{11}}}{\frac{\partial u_1}{\partial q_{12}}} = \frac{\frac{\partial u_2}{\partial q_{21}}}{\frac{\partial u_2}{\partial q_{22}}}$$

$$\Rightarrow \text{MRS}_{Q_1, Q_2} \text{ of consumer I} = \Rightarrow \text{MRS}_{Q_1, Q_2} \text{ of consumer II} \quad (21.16)$$

which is the same as (21.11).

Pareto-efficiency condition (21.11) or (21.16) gives us that the given quantities of the two goods should be distributed among the two consumers in such a way that the MRS between the goods may be the same for the two consumers.

We may now see with the help of a simple example why condition (21.11) is necessary for Pareto efficiency in consumption.

Let us suppose that:

for individual I, $\Rightarrow \text{MRS}_{Q_1, Q_2} = 2$ and

for individual II, $\Rightarrow \text{MRS}_{Q_1, Q_2} = 1$

i.e., the MRS is not the same for the two individuals.

This means that individual I is willing to exchange 2 units of Q_2 for getting 1 unit of Q_1 and individual II is willing to exchange 1 unit of Q_2 for getting 1 unit of Q_1 . In such a case, where the MRS is not the same for the two individuals, we may redistribute the goods to make at least one of them better off without making the other consumer worse off.

What we have to do here is to take away 1 unit of Q_1 from consumer II and give it to I who will give us 2 units of Q_2 in exchange. Now we give one of these units to B to keep his utility level constant—he wants to have 1 unit Q_2 for giving up 1 unit of Q_1 .

But we have now 1 unit of Q_2 left. We can give it to either I or II, and thus make either I or II better off without making the other person worse off. Thus, the initial allocation was not efficient.

The above example shows us that if the MRS of the two individuals are not equal, if the MRS of II is lower, say, than that of I, then we have to take away the marginal unit of good Q_1 from individual II and give it to I whose MRS is higher, and take away from him good Q_2 in exchange.

As we continue the process, the MRS of II would rise as the quantity of Q_1 with him decreases and the MRS of I would decrease as the quantity of Q_1 with him increases, and, as we have seen, the distribution becomes better in the Pareto sense. Therefore, if we are to reach the Pareto-efficient situation, we have to continue the process till MRS of the two persons become equal.

For when the MRS of the two persons are equal, no further redistribution will be able to do good to at least one of them without harming the other. To understand this, let us suppose that MRS of both the persons are equal, and it is equal to 4.

In that case, if we take away 1 unit of Q_1 from consumer II and give it to consumer I, the latter

would give us 4 units of Q_2 in exchange in order to keep his utility level intact. If we now give these 4 units of to individual II, his utility would assume the initial level. That is, by means of a redistribution of the goods, we have not been able to improve the utility level of at least one of the persons. On the contrary, a redistribution of the goods would keep the individuals on their initial utility levels.

5. Pareto Optimality in Consumption or Exchange and Perfect Competition:

It can be easily shown that Pareto optimality in consumption is automatically achieved under perfect competition. For under perfect competition, the prices P_1 and P_2 of the two goods are given to the consumers, and each utility-maximising consumer equates his MRS of Q_1 for Q_2 to the ratio of the prices of the goods.

That is, for consumer I, we get:

$$\left. \begin{array}{l} \frac{\partial u_1}{\partial q_{11}} = \frac{P_1}{P_2} \\ \frac{\partial u_1}{\partial q_{12}} \end{array} \right\} \text{and for consumer II we have} \quad (21.17)$$

$$\left. \begin{array}{l} \frac{\partial u_2}{\partial q_{21}} = \frac{P_1}{P_2} \\ \frac{\partial u_2}{\partial q_{22}} \end{array} \right\}$$

From (2.17), we have

$$\frac{\frac{\partial u_1}{\partial q_{11}}}{\frac{\partial u_1}{\partial q_{12}}} = \frac{\frac{\partial u_2}{\partial q_{21}}}{\frac{\partial u_2}{\partial q_{22}}} \Rightarrow \text{MRS}_{Q_1, Q_2} \text{ of consumer I} = \text{MRS}_{Q_1, Q_2} \text{ of consumer II.} \quad (21.18)$$

which is nothing but the Pareto-efficiency condition (21.16) or (21.11).

Thus, perfect competition guarantees Pareto-efficiency in the distribution of commodities among the consumers.

6. Pareto Optimality Conditions when the External Effects are Present:

The marginal condition for a Pareto-efficient distribution of given amounts of two goods (Q_1 and Q_2) between the two individuals (I and II) as given by (21.18) has been obtained on the basis of the assumption that externalities in consumption are absent.

We shall now see that if the external effects are present, the Pareto optimality condition in consumption would generally be different from the marginal condition (21.18).

Let us assume that the external effects are present in consumption in the sense that the utility level of one consumer depends also on the consumption of another.

Let us assume that the two consumers' utility functions are given by:

$$\begin{aligned} u_1 &= u_1(q_{11}, q_{12}, q_{21}, q_{22}) \\ u_2 &= u_2(q_{11}, q_{12}, q_{21}, q_{22}) \end{aligned} \quad (21.19)$$

where $q_{11} + q_{21} = q_1^0$ and $q_{12} + q_{22} = q_2^0$.

Pareto optimality will be achieved if u_1 is maximum subject to a given level of $u_2 = u_2^0$.

In order to derive the conditions for this constrained maximisation, we have to form the Lagrange function:

$$L = u_1(q_{11}, q_{12}, q_{21}, q_{22}) + \lambda [u_2(q_{11}, q_{12}, q_1^0 - q_{11}, q_2^0 - q_{12}) - u_2^0] \quad (21.21)$$

The first order or necessary conditions for the constrained maximum u_1 are given by

$$\left. \begin{aligned} \frac{\partial L}{\partial q_{11}} &= \frac{\partial u_1}{\partial q_{11}} - \frac{\partial u_1}{\partial q_{21}} + \lambda \left[\frac{\partial u_2}{\partial q_{11}} - \frac{\partial u_2}{\partial q_{21}} \right] = 0 \\ \frac{\partial L}{\partial q_{12}} &= \frac{\partial u_1}{\partial q_{12}} - \frac{\partial u_1}{\partial q_{22}} + \lambda \left[\frac{\partial u_2}{\partial q_{12}} - \frac{\partial u_2}{\partial q_{22}} \right] = 0 \\ \frac{\partial L}{\partial \lambda} &= u_2(q_{11}, q_{12}, q_1^0 - q_{11}, q_2^0 - q_{12}) - u_2^0 = 0 \end{aligned} \right\} \quad (21.22)$$

Now, from the first two equations of (21.22), we have

$$\frac{\frac{\partial u_1}{\partial q_{11}} - \frac{\partial u_1}{\partial q_{21}}}{\frac{\partial u_1}{\partial q_{12}} - \frac{\partial u_1}{\partial q_{22}}} = \frac{\frac{\partial u_2}{\partial q_{11}} - \frac{\partial u_2}{\partial q_{21}}}{\frac{\partial u_2}{\partial q_{12}} - \frac{\partial u_2}{\partial q_{22}}} \quad (21.23)$$

Equation (21.23) is the necessary' condition for Pareto optimality in consumption when external effects are present. It generally differs from the Pareto optimality marginal condition as given by (21.18) or (21.16) or (21.11).

Perfect completion guarantees the attainment of (21.11) but not of (21.23). It is evident from (21.23) that if the external effects were absent, we would have $\partial u_1/\partial q_{21}$, $\partial u_1/\partial q_{22}$, $\partial u_1/\partial q_{11}$ and $\partial u_2/\partial q_{12}$, all equal to zero, and then (21.23) would have reduced to (21.11).

Since we have assumed here that the partial derivatives of the utility functions are functions of all variables, viz., q_{11} , q_{12} , q_{21} and q_{22} , the optimum position of each consumer depends upon the consumption level of the other.

For example, if we assume that the only external effect present in the two-consumer model is $\partial u_2/\partial q_{11} < 0$, then equation (21.23) becomes:

$$\frac{\frac{\partial u_1}{\partial q_{11}}}{\frac{\partial u_1}{\partial q_{12}}} = \frac{\frac{\partial u_2}{\partial q_{11}} - \frac{\partial u_2}{\partial q_{21}}}{-\frac{\partial u_2}{\partial q_{22}}}$$

$$= -\frac{\frac{\partial u_2}{\partial q_{11}}}{\frac{\partial u_2}{\partial q_{22}}} + \frac{\frac{\partial u_2}{\partial q_{21}}}{\frac{\partial u_2}{\partial q_{22}}}$$

i.e., MRS_{Q_1, Q_2} of consumer I = $-\frac{\frac{\partial u_2}{\partial q_{11}}}{\frac{\partial u_2}{\partial q_{22}}} + MRS_{Q_1, Q_2}$ of consumer II. (21.24)

Therefore, (21.24) gives us that, in this case of external effects, MRS_{Q_1, Q_2} of consumer II

should be less than that of consumer I for an optimal distribution, since $\frac{\frac{\partial u_2}{\partial q_{11}}}{\frac{\partial u_2}{\partial q_{22}}} < 0$. It may be

intuitively understood why this is so. If consumer I's consumption of Q_1 increases, then the utility level of consumer II declines. This implies that the marginal significance of Q_1 to consumer II is relatively large, which, again, implies that the MRS_{Q_1, Q_2} of consumer II should be smaller at the state of optimal distribution of the goods.

For, at this distribution as compared with the MRS-equating distribution, the quantity of Q_1 possessed by consumer II would be larger than that possessed by consumer I.

It can be shown diagrammatically with the help of Fig. 21.3 that condition (21.16) does not necessarily ensure Pareto optimality in the presence of external effects. Figures 21.3(a) and 21.3(b) give us the indifference map of consumers I and II, respectively. Let us assume that initially, consumer I consumes the combination A and consumer II consumes the combination E. The MRS_{Q_1, Q_2} of the two consumers are equal at their utility maximising points, given the prices of the goods. Let us now assume that there are no external effects for consumer I, i.e., I's utility level is not affected by the consumption of II.

Although, consumer II's utility level is affected by the consumption of consumer I. Let us suppose, as we have done above, that as I consumes more of Q_1 , II's utility level declines, i.e., $\partial u_2 / \partial q_{11} < 0$. This is the external effect present here.

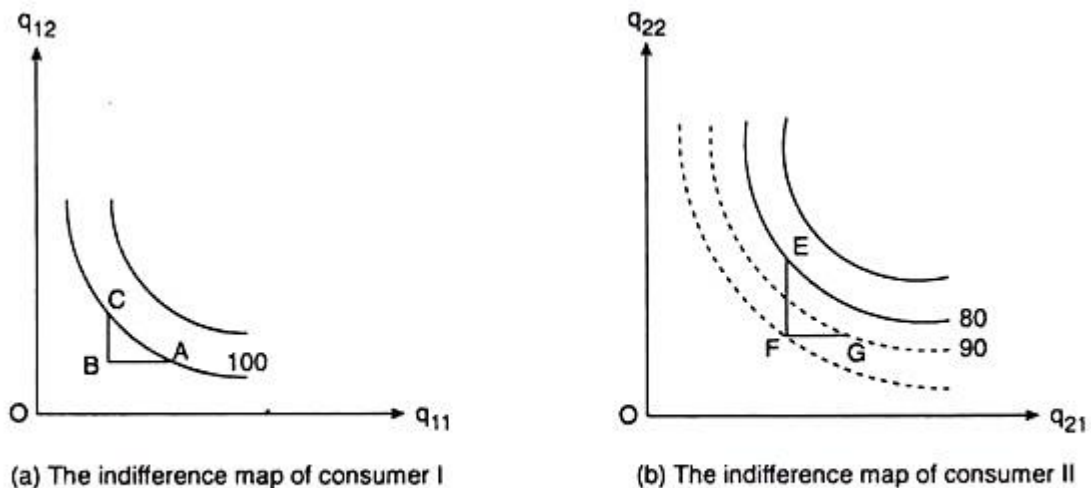


Fig. 21.3 Presence of external effects in consumption

Now, in Fig. 21.3(b), consumer II's indifference curves (solid ones) have been drawn on the assumption that I's consumption is given by combination A. In their individual equilibrium situations, consumer I's utility index is 100 and that of II is 80.

Let us now redistribute the commodities between the two individuals such that their aggregate quantities remain unchanged and I moves to point C having less of Q_1 and more of Q_2 and II moves to point G having more of Q_1 and less of Q_2 ($AB = FG$ and $BC = EF$). The utility level of consumer I has not changed because of this redistribution—he remains on the same IC.

However, since consumer I's consumption of Q_1 has decreased, consumer II's preference-indifference pattern would be affected. His new ICs are given by the dotted curves. Also, at the point G, consumer II's utility level has increased to 90 since I is now consuming less of Q_1 .

Therefore, by means of the redistribution, we have been able to raise II's utility level, I's level remaining constant. That is, the initial equilibrium positions at A and E where the MRS of the consumers had been equal, were Pareto non-optimal. Therefore, we have seen that equality of MRS of the two consumers does not ensure Pareto optimality.

In the present equilibrium situations, the MRS of consumer I has increased since he has moved north-westward along the same IC, and the MRS of II has decreased since he has moved southeastward, not along the same IC, but along an almost parallel IC.

That is, if the said external effect is present, consumer II's MRS would be less than that of consumer I. This result we have already obtained in the mathematical analysis given above.

7. Efficiency in the Allocation of Factors among Commodities, or, Efficiency in Product-Mix or Composition of Output:

A composition of output or product-mix is Pareto-efficient if it is impossible to increase the utility of one individual without reducing the utility of the other by reallocating the factors among the commodities, leading to a different product-mix.

The marginal condition for a Pareto-efficient product-mix states that the marginal rate of product transformation (MRPT) of Q_2 into Q_1 must be the same as the marginal rate of substitution (MRS) of Q_1 for Q_2 , for each consumer.

Here, the MRPT of Q_2 into Q_1 is equal to the quantity by which the production of Q_2 has to be reduced in order to produce one more (or the marginal) unit of Q_1 and, as such, it is equal to the numerical slope of the economy's production possibility curve or frontier (PPC or PPF).

An economy's PPC passes through all the combinations of the two goods (Q_1 and Q_2) that the available quantities of the two inputs (X_1 and X_2) can produce Pareto-efficiently. That is, any combination of the two goods that lie on the PPC gives us the maximum quantity of Q_1 that can be produced subject to the production of a given quantity of Q_2 , or, the maximum of Q_2 subject to a given quantity of Q_1 .

In other words, the combinations of the two goods that lie on the PPC are those that lie on the Edge-worth contract curve for production (CCP) [Fig. 21.1]. That is, there is a one-to-one correspondence between the points on the CCP and those on the PPC. Since, as we move along the CCP, the quantity of one of the goods increases and that of the other decreases, the slope of the PPC would be negative.

Also, as more and more inputs are removed from the production of Q_2 and are engaged in the production of Q_1 , Q_2 may be transformed into Q_1 at a constant rate in which case the PPC would be a negatively sloped straight line with its numerical slope or MRPT being a constant, or, which is more likely, Q_2 may be transformed into Q_1 at an increasing rate owing to the law of diminishing marginal product, in which case the PPC would be concave to the origin with its numerical slope or MRPT rising as Q_1 increases and Q_2 diminishes, i.e., as we move southeastward along the curve. We have shown these two types of PPC in Fig. 21.4.

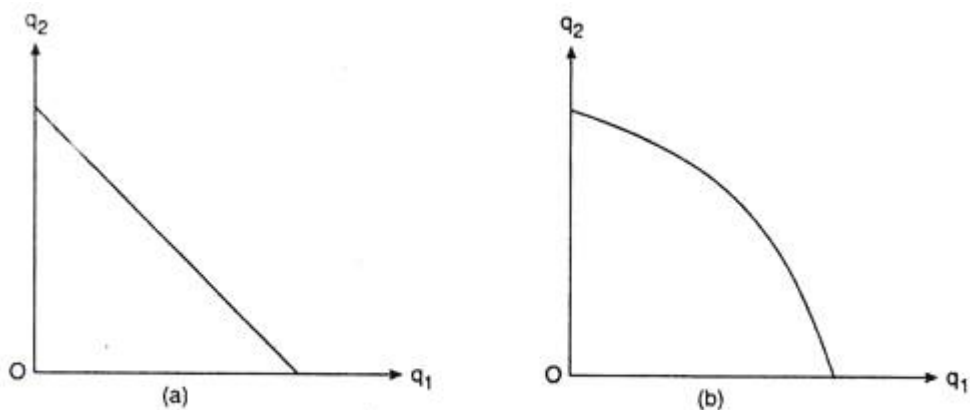


Fig. 21.4 The production possibility curve

Now, since the MRPT shows the rate at which a good can be transformed into another in production and the MRS shows the rate at which the consumers are willing to exchange one good for another, the Pareto-efficient product-mix cannot be obtained unless the two rates are equal. Only then the production sector's plans may be consistent with the household sector's plans, and the two are in equilibrium.

We may illustrate the argument with the help of a simple numerical example. Let us assume that at any particular product composition, the MRPT is 7, i.e., 7 units of Q_2 can be transformed into 1 unit of Q_1 .

On the other hand, at this product composition, the MRS for each consumer is, say, 3. That is, for substituting an additional (or the marginal) unit of Q_1 , each consumer is willing to forego 3 units of Q_2 so that his utility level might remain constant.

To improve the welfare situation, what we may do in this case is: we may take away from each consumer 1 unit of Q_1 and in its place we may have 7 units of Q_2 and then, out of these 7 units, we may give 3 units to the consumer to compensate for his loss of 1 unit of Q_1 . We are then left with 4 units of Q_2 for each consumer.

If their number is 2, then we are left with 8 units of Q_2 some of which we may give to consumer I and some to consumer II. Thus the utility level of both the consumers would increase. This shows us that the initial situation of $MRPT \neq MRS$ was Pareto-non-optimal.

Now, as we take away Q_1 from each consumer, his MRS_{Q_1, Q_2} would increase (from 3) and as we move northwestward along the PPC curve to have Q_2 in its place, the MRPT would decrease (from 7). We have to continue the process unless at some product composition MRPT becomes equal to MRS.

Therefore, the marginal condition for the Pareto-efficient product-mix gives us that the MRPT between the products should be equal to the MRS of each consumer. It may very well be seen that once these two become equal, no improvement in welfare can be achieved by any further change in product composition.

For example, if both MRPT and MRS are equal to 5, say, then, if we take away 1 unit of Q_1 from each consumer, 5 more units of Q_2 would be obtained in its place, and all of these 5 units would have to be given to the consumer to compensate for his loss of 1 unit of Q_1 —to keep him on his initial utility level. Thus, nothing would be available for any improvement.

What is Arrow's Impossibility Theorem?

Arrow's impossibility theorem is a social-choice paradox illustrating the flaws of ranked voting systems. It states that a clear order of preferences cannot be determined while adhering to mandatory principles of fair voting procedures

Understanding Arrow's Impossibility Theorem

Democracy depends on people's voices being heard. For example, when it is time for a **gov**

government to be formed, an election is called, and people head to the polls to vote. Millions of voting slips are then counted to determine who is the most popular candidate and the next elected official.

According to Arrow's impossibility theorem, in all cases where preferences are ranked, it is impossible to formulate a social ordering without violating one of the following conditions:

- **Nondictatorship:** The wishes of multiple voters should be taken into consideration.
- **Pareto Efficiency:** Unanimous individual preferences must be respected: If every voter prefers candidate A over candidate B, candidate A should win.
- **Independence of Irrelevant Alternatives:** If a choice is removed, then the others' order should not change: If candidate A ranks ahead of candidate B, candidate A should still be ahead of candidate B, even if a third candidate, candidate C, is removed from participation.
- **Unrestricted Domain:** Voting must account for all individual preferences.
- **Social Ordering:** Each individual should be able to order the choices in any way and indicate ties.

Arrow's impossibility theorem, part of social choice theory, an economic theory that considers whether a society can be ordered in a way that reflects individual preferences, was lauded as a major breakthrough. It went on to be widely used for analyzing problems in welfare economics.

Example of Arrow's Impossibility Theorem

Let's look at an example illustrating the type of problems highlighted by Arrow's impossibility theorem. Consider the following example, where voters are asked to rank their preference of three projects that the country's annual tax dollars could be used for: A; B; and C. This country has 99 voters who are each asked to rank the order, from best to worst, for which of the three projects should receive the annual funding.

- 33 votes $A > B > C$ (1/3 prefer A over B and prefer B over C)
- 33 votes $B > C > A$ (1/3 prefer B over C and prefer C over A)
- 33 votes $C > A > B$ (1/3 prefer C over A and prefer A over B)

Therefore,

- 66 voters prefer A over B
- 66 voters prefer B over C
- 66 voters prefer C over A

So a two-thirds majority of voters prefer A over B and B over C and C over A---a paradoxical result based on the requirement to rank order the preferences of the three alternatives.

Arrow's theorem indicates that if the conditions cited above in this article i.e. Non-dictatorship, Pareto efficiency, independence of irrelevant alternatives, unrestricted domain, and social ordering are to be part of the decision making criteria then it is impossible to formulate a social ordering on a problem such as indicated above without violating one of the following conditions.

Arrow's impossibility theorem is also applicable when voters are asked to rank political candidates. However, there are other popular voting methods, such as approval voting or plurality voting, that do not use this framework.

UNIT-IV

GENERAL EQUILIBRIUM ANALYSIS

As against partial equilibrium analysis, general equilibrium analysis is concerned with economic system as a whole. It recognises the fact that economic system is a network in which all the parts are mutually dependent on one another and in mutual interaction with one another.

Goods are either competitive or substitutes. Some goods are used in the manufacture of other goods. Factors of production are complementary to each other to the extent they can be substituted for each other, they are competitive also. Resources also face competitive demand from producers. Therefore, change in the demand or supply of any commodity or factor of production sets in motion a chain reaction. A disturbance in one sector of the economy produces its repercussions on all sides. General equilibrium analysis is concerned with the overall effects of a disturbance. Instead of taking only a few variables at a time, we take into consideration all the relevant variables which may affect the particular phenomenon in hand. In this type of analysis, all the side-effects of an economic disturbance are analysed in full.

An example will make the concept of general equilibrium clearer. Suppose the demand for India-manufactured consumer goods suddenly increases in Western Europe. Indian exports will increase thereby increasing output, employment and profits in the export industries. Resources will be diverted from other industries to the export industries. The demand and prices of the substitute commodities will also increase. The increased demand for exports will have economy-wide effects.

An all-round analysis of the repercussions of the economic disturbance increased demand for manufactured consumer goods for export can be done only through general equilibrium theory.

General equilibrium analysis deals with the equilibrium of the whole organisation in the economy consumers, producers, resource-owners, firms and industries. Not only should individual consumers and firms be in equilibrium in themselves but also in relation to each other. Business firms enter product markets as suppliers, but they enter factor markets as buyers. Households, on the other hand, are buyers in product markets but suppliers in factor markets. General equilibrium prevails when both the product and factor markets are in equilibrium in relation to each other.

Objectives of General Equilibrium Analysis:

General equilibrium analysis serves many important purposes.

Firstly, it provides us with a theoretical tool to understand the economy in its entirety the mechanics of its working, its structure, and the major forces making it work. The theory is an analysis of the interrelationships of the various sectors of an economy. As such, it helps us in knowing clearly the economy-wide implications of an economic change.

Secondly, we can apply general equilibrium theory to determine the primary, secondary and tertiary effects of an economic disturbance which has an intersectoral impact. Whenever there is an economic disturbance say, like the defence programmes in the wake of Chinese aggression in 1962 it has some immediate effects in one sector of the economy. Gradually, the impact of such a disturbance is felt in other sectors. The whole economy goes into disequilibrium. Process of adjustment to the economic disturbances starts to establish a new equilibrium.

As Richard Leftwich put it, "First comes the big splash from the disturbance. Particular equilibrium analysis handles the splash. But waves and then ripples are set up from it, affecting one another and affecting the area of the splash. The ripples run farther and farther, becoming smaller and smaller, until eventually they

dwindle away. The tools of general equilibrium are required for analysis of the entire series of readjustments.”

Thus, general equilibrium theory is of great value in stressing the interdependence of various parts of the economic system, which is easily lost sight of in the use of partial equilibrium theory in micro-economic analysis. Failure to recognise this interdependence is responsible for many errors in popular reasoning on economic policy.

Some Key Highlights

- General equilibrium analysis is useful when there is strong inter-relationship between commodities or factors.
- General equilibrium analysis considers simultaneous equilibrium of all markets.
- This analysis is useful when changes in conditions in one market have significant repercussions on other market.
- Economic system as whole is inter-dependent and inter-related .
- There are large number of decision making agents- consumers, producers, workers etc. All these agents self interested and would behave to maximize their goals.
- General equilibrium would occur when markets for all commodities and factors and all decision making agents- consumers, producers, resource owners are simultaneously in equilibrium.
- Partial equilibrium analysis ignores the inter-relations or inter- dependence between prices of commodities and factor of productions.
- In partial analysis each firm is considered independent or self contained.
- This analysis is not useful to apply when there is strong inter-relationship between commodities or factors.

- Partial analysis is useful when the changes in conditions in one market have little repercussions on other markets.

To sum up, partial equilibrium analysis focuses on explaining the determination of price and quantity in a given product or factor market when one market is viewed as independent of other markets. General equilibrium analysis on other hand deals with explaining simultaneous equilibrium in all markets when prices and quantities of all products and factors are considered variable.

Walrasian equilibrium

The first attempt in neoclassical economics to model prices for a whole economy was made by Léon Walras. Walras' *Elements of Pure Economics* provides a succession of models, each taking into account more aspects of a real economy (two commodities, many commodities, production, growth, money). Some think Walras was unsuccessful and that the later models in this series are inconsistent.^{[2][3]}

In particular, Walras's model was a long-run model in which prices of capital goods are the same whether they appear as inputs or outputs and in which the same rate of profits is earned in all lines of industry. This is inconsistent with the quantities of capital goods being taken as data. But when Walras introduced capital goods in his later models, he took their quantities as given, in arbitrary ratios. (In contrast, Kenneth Arrow and Gérard Debreu continued to take the initial quantities of capital goods as given, but adopted a short run model in which the prices of capital goods vary with time and the own rate of interest varies across capital goods.)

Walras was the first to lay down a research program much followed by 20th-century economists. In particular, the Walrasian agenda included the investigation of when equilibria are unique and stable—Walras' Lesson 7 shows neither uniqueness, nor stability, nor even existence of an equilibrium is guaranteed. Walras also proposed a dynamic process by which general equilibrium might be reached, that of the tâtonnement or groping process.

The tâtonnement process is a model for investigating stability of equilibria. Prices are announced (perhaps by an "auctioneer"), and agents state how much of each

good they would like to offer (supply) or purchase (demand). No transactions and no production take place at disequilibrium prices. Instead, prices are lowered for goods with positive prices and excess supply. Prices are raised for goods with excess demand. The question for the mathematician is under what conditions such a process will terminate in equilibrium where demand equates to supply for goods with positive prices and demand does not exceed supply for goods with a price of zero. Walras was not able to provide a definite answer to this question

Marshall and Sraffa

In partial equilibrium analysis, the determination of the price of a good is simplified by just looking at the price of one good, and assuming that the prices of all other goods remain constant. The Marshallian theory of supply and demand is an example of partial equilibrium analysis. Partial equilibrium analysis is adequate when the first-order effects of a shift in the demand curve do not shift the supply curve. Anglo-American economists became more interested in general equilibrium in the late 1920s and 1930s after Piero Sraffa's demonstration that Marshallian economists cannot account for the forces thought to account for the upward-slope of the supply curve for a consumer good.

If an industry uses little of a factor of production, a small increase in the output of that industry will not bid the price of that factor up. To a first-order approximation, firms in the industry will experience constant costs, and the industry supply curves will not slope up. If an industry uses an appreciable amount of that factor of production, an increase in the output of that industry will exhibit increasing costs. But such a factor is likely to be used in substitutes for the industry's product, and an increased price of that factor will have effects on the supply of those substitutes. Consequently, Sraffa argued, the first-order effects of a shift in the demand curve of the original industry under these assumptions includes a shift in the supply curve of substitutes for that industry's product, and consequent shifts in the original industry's supply curve. General equilibrium is designed to investigate such interactions between markets.

Continental European economists made important advances in the 1930s. Walras' proofs of the existence of general equilibrium often were based on the counting of

equations and variables. Such arguments are inadequate for non-linear systems of equations and do not imply that equilibrium prices and quantities cannot be negative, a meaningless solution for his models. The replacement of certain equations by inequalities and the use of more rigorous mathematics improved general equilibrium modeling.

Modern concept of general equilibrium in economics

The modern conception of general equilibrium is provided by a model developed jointly by Kenneth Arrow, Gérard Debreu, and Lionel W. McKenzie in the 1950s.^{[4][5]} Debreu presents this model in *Theory of Value* (1959) as an axiomatic model, following the style of mathematics promoted by Nicolas Bourbaki. In such an approach, the interpretation of the terms in the theory (e.g., goods, prices) are not fixed by the axioms.

Three important interpretations of the terms of the theory have been often cited. First, suppose commodities are distinguished by the location where they are delivered. Then the Arrow-Debreu model is a spatial model of, for example, international trade.

Second, suppose commodities are distinguished by when they are delivered. That is, suppose all markets equilibrate at some initial instant of time. Agents in the model purchase and sell contracts, where a contract specifies, for example, a good to be delivered and the date at which it is to be delivered. The Arrow-Debreu model of intertemporal equilibrium contains forward markets for all goods at all dates. No markets exist at any future dates.

Third, suppose contracts specify states of nature which affect whether a commodity is to be delivered: "A contract for the transfer of a commodity now specifies, in addition to its physical properties, its location and its date, an event on the occurrence of which the transfer is conditional. This new definition of a commodity allows one to obtain a theory of [risk] free from any probability concept..."^[6]

These interpretations can be combined. So the complete Arrow-Debreu model can be said to apply when goods are identified by when they are to be delivered, where they are to be delivered and under what circumstances they are to be delivered, as

well as their intrinsic nature. So there would be a complete set of prices for contracts such as "1 ton of Winter red wheat, delivered on 3rd of January in Minneapolis, if there is a hurricane in Florida during December". A general equilibrium model with complete markets of this sort seems to be a long way from describing the workings of real economies, however its proponents argue that it is still useful as a simplified guide as to how real economies function.

Some of the recent work in general equilibrium has in fact explored the implications of incomplete markets, which is to say an intertemporal economy with uncertainty, where there do not exist sufficiently detailed contracts that would allow agents to fully allocate their consumption and resources through time. While it has been shown that such economies will generally still have an equilibrium, the outcome may no longer be Pareto optimal. The basic intuition for this result is that if consumers lack adequate means to transfer their wealth from one time period to another and the future is risky, there is nothing to necessarily tie any price ratio down to the relevant marginal rate of substitution, which is the standard requirement for Pareto optimality. Under some conditions the economy may still be constrained Pareto optimal, meaning that a central authority limited to the same type and number of contracts as the individual agents may not be able to improve upon the outcome, what is needed is the introduction of a full set of possible contracts. Hence, one implication of the theory of incomplete markets is that inefficiency may be a result of underdeveloped financial institutions or credit constraints faced by some members of the public. Research still continues in this area.

Basic questions in general equilibrium analysis are concerned with the conditions under which an equilibrium will be efficient, which efficient equilibria can be achieved, when an equilibrium is guaranteed to exist and when the equilibrium will be unique and stable.

First Fundamental Theorem of Welfare Economics

The First Fundamental Welfare Theorem asserts that market equilibria are Pareto efficient. In a pure exchange economy, a sufficient condition for the first welfare theorem to hold is that preferences be locally nonsatiated. The first welfare theorem also holds for economies with production regardless of the properties of the production function. Implicitly, the theorem assumes complete markets and

perfect information. In an economy with externalities, for example, it is possible for equilibria to arise that are not efficient.

The first welfare theorem is informative in the sense that it points to the sources of inefficiency in markets. Under the assumptions above, any market equilibrium is tautologically efficient. Therefore, when equilibria arise that are not efficient, the market system itself is not to blame, but rather some sort of market failure.

Second Fundamental Theorem of Welfare Economics

Even if every equilibrium is efficient, it may not be that every efficient allocation of resources can be part of an equilibrium. However, the second theorem states that every Pareto efficient allocation can be supported as an equilibrium by some set of prices. In other words, all that is required to reach a particular Pareto efficient outcome is a redistribution of initial endowments of the agents after which the market can be left alone to do its work. This suggests that the issues of efficiency and equity can be separated and need not involve a trade-off. The conditions for the second theorem are stronger than those for the first, as consumers' preferences and production sets now need to be convex (convexity roughly corresponds to the idea of diminishing marginal rates of substitution i.e. "the average of two equally good bundles is better than either of the two bundles").

Existence

Even though every equilibrium is efficient, neither of the above two theorems say anything about the equilibrium existing in the first place. To guarantee that an equilibrium exists, it suffices that consumer preferences be strictly convex. With enough consumers, the convexity assumption can be relaxed both for existence and the second welfare theorem. Similarly, but less plausibly, convex feasible production sets suffice for existence; convexity excludes economies of scale.

Proofs of the existence of equilibrium traditionally rely on fixed-point theorems such as Brouwer fixed-point theorem for functions (or, more generally, the Kakutani fixed-point theorem for set-valued functions). See Competitive equilibrium#Existence of a competitive equilibrium. The proof was first due to Lionel McKenzie,^[7] and Kenneth Arrow and Gérard Debreu.^[8] In fact, the converse also holds, according to Uzawa's derivation of Brouwer's fixed point theorem from Walras's law.^[9] Following Uzawa's theorem, many mathematical

economists consider proving existence a deeper result than proving the two Fundamental Theorems.

Another method of proof of existence, global analysis, uses Sard's lemma and the Baire category theorem; this method was pioneered by Gérard Debreu and Stephen Smale.

Uniqueness

Although generally (assuming convexity) an equilibrium will exist and will be efficient, the conditions under which it will be unique are much stronger. The Sonnenschein–Mantel–Debreu theorem, proven in the 1970s, states that the aggregate excess demand function inherits only certain properties of individual's demand functions, and that these (Continuity, Homogeneity of degree zero, Walras' law and boundary behavior when prices are near zero) are the only real restriction one can expect from an aggregate excess demand function. Any such function can represent the excess demand of an economy populated with rational utility-maximizing individuals.

There has been much research on conditions when the equilibrium will be unique, or which at least will limit the number of equilibria. One result states that under mild assumptions the number of equilibria will be finite (see regular economy) and odd (see index theorem). Furthermore, if an economy as a whole, as characterized by an aggregate excess demand function, has the revealed preference property (which is a much stronger condition than revealed preferences for a single individual) or the gross substitute property then likewise the equilibrium will be unique. All methods of establishing uniqueness can be thought of as establishing that each equilibrium has the same positive local index, in which case by the index theorem there can be but one such equilibrium.

Determinacy

Given that equilibria may not be unique, it is of some interest to ask whether any particular equilibrium is at least locally unique. If so, then comparative statics can be applied as long as the shocks to the system are not too large. As stated above, in

a regular economy equilibria will be finite, hence locally unique. One reassuring result, due to Debreu, is that "most" economies are regular.

Work by Michael Mandler (1999) has challenged this claim.^[17] The Arrow– Debreu– McKenzie model is neutral between models of production functions as continuously differentiable and as formed from (linear combinations of) fixed coefficient processes. Mandler accepts that, under either model of production, the initial endowments will not be consistent with a continuum of equilibria, except for a set of Lebesgue measure zero. However, endowments change with time in the model and this evolution of endowments is determined by the decisions of agents (e.g., firms) in the model. Agents in the model have an interest in equilibria being indeterminate:

Indeterminacy, moreover, is not just a technical nuisance; it undermines the price- taking assumption of competitive models. Since arbitrary small manipulations of factor supplies can dramatically increase a factor's price, factor owners will not take prices to be parametric.^{[17]:17}

When technology is modeled by (linear combinations) of fixed coefficient processes, optimizing agents will drive endowments to be such that a continuum of equilibria exist:

The endowments where indeterminacy occurs systematically arise through time and therefore cannot be dismissed; the Arrow-Debreu-McKenzie model is thus fully subject to the dilemmas of factor price theory.^{[17]:19}

Some have questioned the practical applicability of the general equilibrium approach based on the possibility of non-uniqueness of equilibria.

Stability

In a typical general equilibrium model the prices that prevail "when the dust settles" are simply those that coordinate the demands of various consumers for various goods. But this raises the question of how these prices and allocations have been arrived at, and whether any (temporary) shock to the economy will cause it to converge back to the same outcome that prevailed before the shock. This is the question of stability of the equilibrium, and it can be readily seen that it is related to the question of uniqueness. If there are multiple equilibria, then some of them will be unstable.

