ABERRATIONS

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Aberrations

Defects in images formed by a lens or combination of lenses are known as Aberrations i.e. the deviations from the actual size, shape, position and colour of image produced by a lens in comparison to object are called aberrations. They are divided into two types. They are

- Chromatic Aberration
- Monochromatic Aberrations

Aberrations

Chromatic Aberration is due to the dispersion of white light. If monochromatic light is used this defect is eliminated. But there are some other defects in images even if monochromatic light is used. These defects are known as "Monochromatic Aberrations".

They are

- Spherical Aberration
- Coma
- Astigmatism
- Curvature
- Distortion

Chromatic Aberration

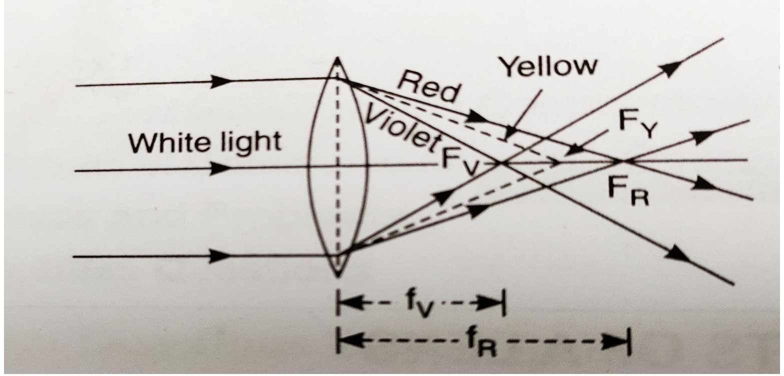
The aberration produced by the variation of refractive index with wavelength of light is called Chromatic Aberration.

When white light is incident on a convex lens it splits up into seven colours .

This is known as Dispersion.

We know that focal length f of the lens is given by

$$1/f = (\mu-1)(1/R_1 - 1/R_2).$$

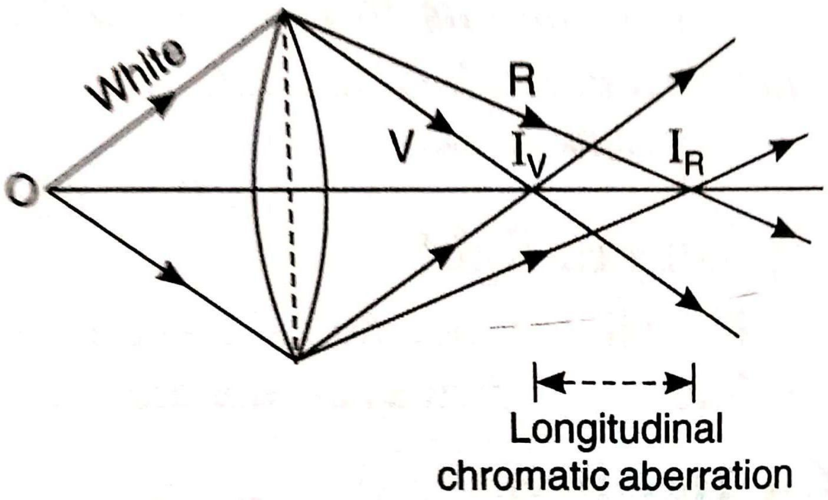


- The refractive index (μ) changes from colour to colour .
- The Refractive index for violet (μ_V) is greater than that of Refractive index of red (μ_R)

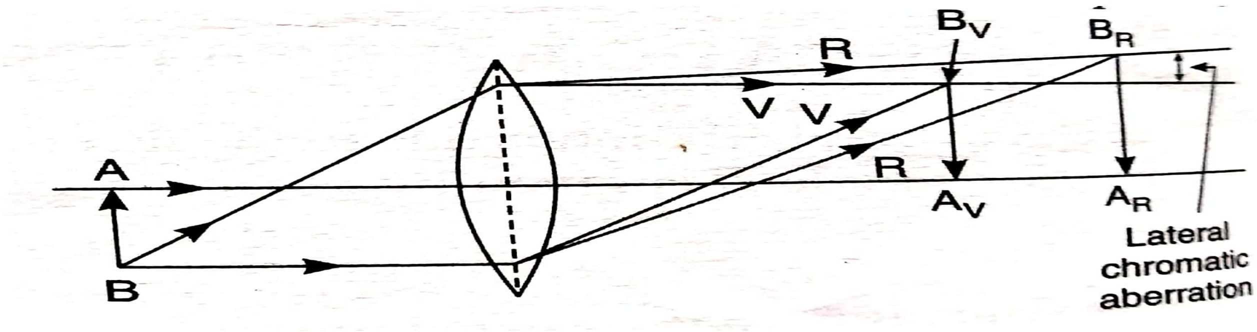
$$\text{i.e.} \quad \mu_V > \mu_R .$$

- Therefore the focal length of red rays (f_R) is greater than the focal length of violet rays (f_V) . i.e. different coloured images are brought to focus at different positions on the axis .
- Thus the image formed by the lens for a parallel beam of white light appears to be blurred.
- This defect in lenses is known as **"Longitudinal Chromatic Aberration"**.
- The difference (f_R - f_V) is the measure of longitudinal chromatic aberration .

- when white point object O is situated on the axis of the lens then violet and red images I_V , I_R are as shown in figure.
- Longitudinal Chromatic Aberration = $I_V - I_R$



- Because focal length changes from colour to colour the images of different colours have different magnifications .
- This defect is known as “ **lateral Chromatic Aberration”.**



- These two defects longitudinal and Lateral Chromatic Aberration are combinedly known as **“Chromatic Aberration”.**

EXPRESSION FOR LONGITUDINAL CHROMATIC ABERRATION:

Let f_R, f_V, f are focal length for red, violet and mean ray respectively and μ_V, μ_R, μ are respective refractive indices then

$$\frac{1}{f_R} = (\mu_R - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{-------}(1)$$

$$\frac{1}{f_V} = (\mu_V - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{-----}(2)$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{--------}(3)$$

Subtracting 2nd equation from 1st equation we get

$$\frac{1}{f_V} - \frac{1}{f_R} = (\mu_V - \mu_R)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{f_R - f_V}{f_R f_V} = (\mu_V - \mu_R)\left(\frac{1}{R_1} - \frac{1}{R_1}\right)$$

$$\frac{f_R - f_V}{f_R f_V} = \frac{(\mu_v - \mu_n)}{(\mu - 1)}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)(\mu - 1)$$

$$\frac{f_R - f_V}{f^2} = \frac{(\mu_v - \mu_s)}{(\mu - 1)}\left(\frac{1}{f}\right) \quad \text{from } (3)(\because f_R f_V = f^2)$$

$$\left(\frac{f_R - f_V}{f}\right) = \frac{\mu_v - \mu_n}{\mu - 1}$$

$$f_R - f_V = \omega f$$

Hence the Longitudinal Chromatic Aberration for a thin lens in case of an object at infinity is equal to product of dispersive power and focal length of mean ray

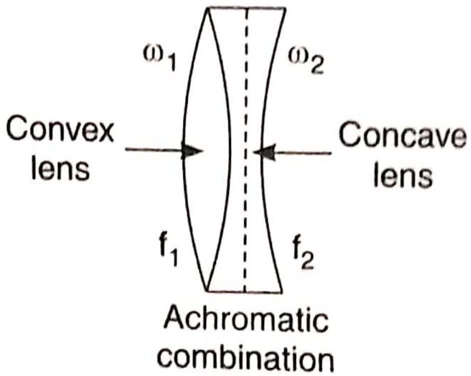
Elimination

- Elimination or minimization of Chromatic Aberration is known as Achromatism.
- When two or more lenses are combined together in such a way, that the combination is free from chromatic Aberration, then the combination is known as **Achromatic Combination**.
- Chromatic Aberration can be eliminated by the following methods
  - By an Achromatic Doublet
  - By the lenses of same material separated by a distance

Elimination

Achromatism for two lenses in contact (Achromatic Doublet)

- Achromatic Doublet is the combination of two lenses in contact with each other which produces no Chromatic Aberration.
- It consists of two lenses a convex lens(crown glass) of small focal length and concave lens (flint glass) of large focal length placed in contact.
- An achromatic doublet shows no colours i.e. all colours come to focus at one point.
- That means all colours have same focal length so the focal length of an Achromatic Doublet is independent of refractive index.



Elimination

Consider two lenses of focal lengths f_1 , f_2 and dispersive powers ω_1 , ω_2. If f is the combined focal length of the system, then

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

differentiating,

$$d(\frac{1}{f}) = d(\frac{1}{f_1}) + d(\frac{1}{f_2})$$

$$(\frac{-1}{f^2})df = (\frac{-df_1}{f_1^2}) + d(\frac{-df_2}{f_2^2})$$

$$(\frac{-1}{f^2})df = (\frac{-df_1}{f_1})\frac{1}{f_1} + d(\frac{-df_2}{f_2})\frac{1}{f_2}$$

Elimination

for no chromatic aberration, df=0 since f is constant, i.e. focal length is the same for all colours.

$$\Rightarrow (\frac{-df_1}{f_1})\frac{1}{f_1} + (\frac{-df_2}{f_2})\frac{1}{f_2} = 0$$

$$\Rightarrow (\frac{\omega_1}{f_1}) + (\frac{\omega_2}{f_2}) = 0 \quad (\because \frac{-df_1}{f_1} = \omega_1, \frac{-df_2}{f_2} = \omega_2)$$

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{-f_1}{f_2}}$$

Since ω_1 , ω_2 are positive f_1,f_2 must have opposite sign i.e. if one lens is convex other should be concave. The material of the two lenses must be different.

Elimination

Achromatism for two lenses separated by a distance:

Consider two lenses of focal lengths f_1, f_2 separated by distance 'x'. The material of the two lenses is the same. Therefore the dispersive power of the two lenses are also the same $\omega_1 = \omega_2 = \omega$.

If 'f' is the combined focal length of the system then

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$$

Differentiating

$$d(\frac{1}{f}) = d(\frac{1}{f_1}) + d(\frac{1}{f_2}) - d(\frac{x}{f_1 f_2})$$

$$\frac{-1}{f^2}df = \frac{-1}{f_1^2}df_1 - \frac{1}{f_2^2}df_2 - x[\frac{1}{f_1}(\frac{-df_1}{f_1^2}) + \frac{1}{f_2}(\frac{-df_2}{f_2})]$$

$$\frac{-1}{f^2}df = \frac{1}{f_1}(\frac{-df_1}{f_1}) + \frac{1}{f_2}(\frac{-df_2}{f_2}) - x[\frac{1}{f_1 f_2}(\frac{-df_1}{f_1}) + \frac{1}{f_1 f_2}(\frac{-df_2}{f_2})]$$

$$\frac{-1}{f^2}df = \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} - \frac{x}{f_1 f_2}[\omega_1 + \omega_2]$$

Elimination

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} - \frac{x}{f_1 f_2}[\omega_1 + \omega_2] = 0$$

Since $\omega_1 = \omega_2 = \omega$

$$\frac{\omega}{f_1} + \frac{\omega}{f_2} - \frac{x}{f_1 f_2}[2\omega] = 0$$

$$\frac{1}{f_1} + \frac{1}{f_2} - \frac{2x}{f_1 f_2} = 0$$

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{2x}{f_1 f_2}$$

$$f_1 + f_2 = 2x$$

$$x = \frac{f_1 + f_2}{2}$$

Thus the condition for achromatism when two lenses are separated by a distance is

$$x = \frac{f_1 + f_2}{2}$$

i.e. for no chromatic aberration the distance between the two lenses must be equal to the mean of the focal lengths of the two lenses.