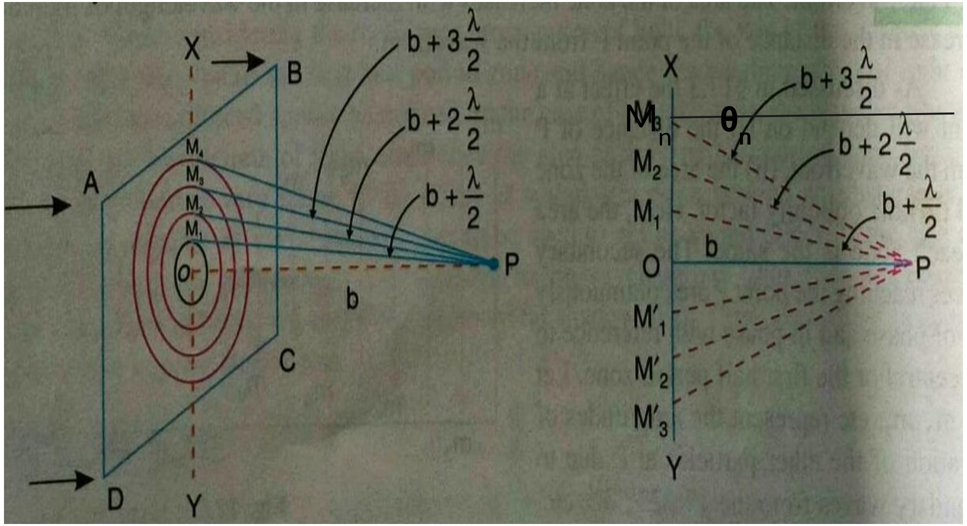
Fresnel's Half Period Zones and Rectilinear Propagation of light

M.SATYA VANI
LECTURER IN PHYSICS
D N R COLLEGE BHIMAVARAM

- Let ABCD be a plane wavefront of wave length λ perpendicular to the plane of the paper.
- P is a point where the resultant intensity is to be calculated. For this purpose Fresnel divided the wavefront into no of zones.
- These are known as Fresnel's half period Zones.



Construction

- Draw a perpendicular OP to the wave front , let the point OP be b.
- The point O is called the pole of the wavefront, with P as center and radii equal to $b + \lambda/2$, $b + 2\lambda/2$, $b + 3\lambda/2$, …….. , $b + n\lambda/2$ draw spheres.
- The plane ABCD cuts these spheres in concentric circles with center O and radii OM_1, OM_2, ……, OM_n .
- The area of first innermost circle is the 1st half period zone, similarly the areas enclosed between first and second circles, second and third circles , ……. (n-1)th and nth circles are known as 2nd,3rd , …… , nth half period zones respectively.
- Each zone differs from its neighbouring zone by a phase difference of π or a path difference of $\lambda/2$ or by a half period (T/2). So they are called half period zones.

Radii and Area of Zone

Radius of 1st half period zone

$$OM_1 = \sqrt{(M_1P)^2 - (OP)^2}$$
$$= \sqrt{(b+\lambda/2)^2 - b^2}$$
$$= \sqrt{b^2 + \lambda^2/4 + b\lambda - b^2}$$
$$= \sqrt{b\lambda} \qquad [\because \lambda \text{ is very small } \lambda^2 \text{ is negligible}]$$

Radius of 2nd half period zone =

$$OM_2 = \sqrt{(M_1P)^2 - (OP)^2}$$
$$= \sqrt{(b+\lambda/2)^2 - b^2}$$
$$= \sqrt{2b\lambda}$$

Similarly the radius of the n^{th} half period zone = $OM_n = \sqrt{nb\lambda}$

Thus the radii of half period zones are proportional to the square root of natural no’s

Radii and Area of Zone

Area of 1st half period zone = $\pi\,(OM_1)^2 = \pi b\lambda$

Area of 2nd half period zone = $\pi\,(OM_2)^2 - \pi\,(OM_1)^2$

$$= 2\pi b\lambda - \pi b\lambda = \pi b\lambda$$

In the same way area of nth half period zone $= \pi\,(OM_n)^2 - \pi\,(OM_{n-1})^2$

$$= \pi b\lambda$$

Thus the area of each half period zone $= \pi b\lambda$

Resultant Intensity at P

The amplitude of disturbance at P due to the wave from a zone varies

1. Directly as the area of the zone
2. Inversely as the distance of P from the wavefront
3. Directly with the obliquity factor

Area of the Zone:

- Area of each half period zone = $\pi b\lambda$
- Since the area is independent of n, all zones have the same area. They send equal no of waves to the point P

Average distance of a zone:

As the order of the Half Period Zones increases the distance of the zone from P increases so the amplitude decreases

Obliquity Factor (θ_n):

This is the angle between the normal to the zone and the line joining the zone to P. The obliquity factor is denoted by f(θ_n).

As the order of the zone increases f(θ_n) decreases .Thus, the amplitude of wave from a zone at P decreases as n increases.

RESULTANT AMPLITUDE:

Let R_1 , R_2 , R_3 .... Represent the amplitudes of the waves at P due to the secondary waves from the second and so on half period zones respectively. The path difference for any two consecutive zones from P differs by $\lambda/2$ which corresponds to a phase difference of π. Thus if the amplitude due to 1st zone is positive, it is negative due to 2nd zone.

Thus the resultant amplitude at P due to the entire wave front

$$= R_1 - R_2 + R_3 - R_4 \ldots\ldots\ldots \pm R_n \text{ (if n is even last term is } -A_n \text{ viceversa.)}$$

As the successive amplitudes R_1,R_2,R_3,R_4 .... are gradually decreasing R_1 is slightly greater than R_2, $R_2 > R_3$ .. ..

$$R_2 = (R_1+R_3)/2 = R_1/2 + R_3/2$$

$$R_4 = (R_3+R_5)/2 = R_3/2 + R_5/2 \ldots\ldots\ldots$$

$$R = R_1/2 + (R_1/2 - R_2 + R_3/2) + (R_3/2 - R_4 + R_5/2) + \ldots\ldots\ldots + (R_{n-2}/2 - R_{n-1} + R_n/2) + R_n/2 \quad \text{n is odd}$$

$$\Rightarrow \quad R = R_1/2 + R_n/2 \text{ when n is odd}$$

$$R = R_1/2 + R_{n-1}/2 - R_n \text{ when n is even}$$

when $n \to \infty$ $R_n, R_{n-1} \to 0$ ($\because$ the amplitudes are gradually decreasing)

$\therefore$ Resultant Amplitude at P due to whole wavefront $= R = R_1/2$

The resultant intensity $I \propto R^2$ i.e. $I \propto R^2/4$

Hence intensity at P is only $(1/4)^{th}$ that of due to first half period zone.

Rectilinear propagation of light:

If a small object is placed at o, it blocks no of first few half period zones . Hence the intensity at **p** will be negligible and practically no light will be received at **p**. If the size of the obstacle is placed in the path of the light is comparable to the wavelength of light used, then it is possible to observe illumination in the region of geometrical shadow also. I.e., rectilinear propagation of light is only approximately true.