Zone Plate

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Zone Plate

It is a specially constructed diffracting screen such that light from every alternate zone is obstructed.

Construction:

- To construct a zone plate, a large no of concentric circles with radii proportional to square root of n natural numbers is drawn on a sheet of white paper. The alternate zones are painted black.
- A highly reduced photograph of this pattern is taken on a thin glass plate.
- This resulting glass negative is known as zone plate.
- There are two types of zone plates, one having odd zones transparent and even zones opaque, where as in other the odd zones are opaque and even zones are transparent
- If the central zone is transparent, the zone plate is called positive and if it is opaque, it is called negative

- Consider a Zone plate AB perpendicular to the plane of the paper.
- S is a monochromatic source of light of wavelength λ
- P is the position of a screen for a bright image
- SO = a and OP = b with O as center and radii equal to $OM_1, OM_2, \ldots, OM_n (r_1, r_2, \ldots, r_n)$
- Divide the plate AB into half period zones such that from one zone to next in a increasing path diff of $\lambda/2$ i.e,

$$SM_1 + M_1P = SO + OP + \lambda/2$$
$$SM_2 + M_2P = SO + OP + 2\lambda/2$$
$$\ldots\ldots\ldots\ldots$$
$$SM_n + M_nP = SO + OP + n\lambda/2 \quad \text{--------} \ (1)$$

From Triangle SM_nO,

$$SM_n = \sqrt{SO^2 + OM^2_n}$$

$$SM_n = \sqrt{a^2 + r^2_n} = a(1+r^2_n/a^2)^{1/2}$$

$$= a(1+r^2_n/2a^2)$$

$$= a + r^2_n/2a \quad \text{----------} \; (2)$$

Similarly from triangle OM_nP,

$$M_nP = \sqrt{OP^2 + OM^2_n}$$

$$M_nP = \sqrt{b^2 + r^2_n} = b(1+r^2_n/2b^2)^{1/2}$$

$$M_nP = b(1+r^2_n/2b^2)$$

$$= b + r^2_n/2b \quad \text{-------------} \; (3)$$

Substituting eq (2) and eq (3) in eq (1)

$$SM_n + M_nP = SO + OP + n\lambda/2$$

$$a + r^2_n/2a + b + r^2_n/2b = a + b + n\lambda/2$$

$$r^2_n/2[1/a + 1/b] = n\lambda/2$$

$$1/a + 1/b = n\lambda/r^2_n = 1/(r^2_n/n\lambda)$$

$$1/f = n\lambda/r^2_n$$

∵ Since this equation is similar to lens maker formula $1/u + 1/v = 1/f$

This is the focal lengths of a zone plate. Thus the zone plate behaves like a convex lens of multiple foci.

- This is the focal lengths of a zone plate. Thus the zone plate behaves like a convex lens of multiple foci
- Let $A_1, A_2, A_3, \ldots\ldots\ldots, A_n$ be the amplitudes due to 1,2,... n^{th} zone respectively. Then resultant amplitude at P is given by

$$A = A_1 + A_3 + A_5 + \ldots\ldots\ldots)$$

- if even zones are opaque then the resultant amplification many times greater than the resultant Amplitude is many times greater than resultant Amplitude $A_1/2$ due to whole wavefront so the resultant intensity at p is much greater than that when the whole wave front is exposed to the point p
- If the parallel beam of white light is incident on a zone plate different colours come to focus at different points along OP. Thus the function of a zone plate is similar to a convex lens

Multiple foci of a zone plate

A zone plate has a no of foci when s is at infinity i.e, a = ∞ then

$$b = r^2_n / n\lambda$$

$$[\because 1/a + 1/b = n\lambda / r^2_n \Rightarrow 0 + 1/b = n\lambda / r^2_n \Rightarrow b = r^2_n / n\lambda]$$

= principal focal length f

If n = 1 then $f_1 = r^2_n / \lambda$

For this focal length each actual zone of the plate contains one half period element and the focal point is P_1 then the intensity of the point is max.

If each zone contains three half period elements with respect to some other point P_3, the alternate three half period elements being obstructed then again the intensity will be maximum. Then the point P_3 is called 2nd focal point then the resultant intensity

$$A_3 = (m_1 - m_2 + m_3) + (m_7 - m_8 + m_9) + \ldots\ldots..$$
$$= \text{half of } (m_1 + m_3 + m_7 + \ldots\ldots)$$
$$\Rightarrow A_3 < A_1$$

Where m_1, m_3, … are roughly 1.3 rd of A_1, A_2, ….

$$F_3 = r^2_n / 3n\lambda = b/3 \text{ or } f_1/3$$

Similarly 3rd focus occurs at a position P_5 when each zone of the plate contains 5 half period elements

$$F_5 = r^2_n / 5n\lambda = f_1/5$$

Thus azone plate has multiple foci given by

$$F_1 = r^2_n / n\lambda, \; F_3 = r^2_n / 3n\lambda, \; F_5 = r^2_n / 5n\lambda \; \ldots\ldots\ldots\ldots\ldots$$

Generally

$$F = r^2_n / (2p+1)n\lambda$$

Comparison between a zone plate & Convex lens

Similarities:

- Both from real images of an object on the side other than the object. The distances of the object of the image are connected together by similar formula in both the cases.

  $1/u + 1/v = 1/f$ --- lens

  $1/a + 1/b = n\lambda / r^2_n$ Zone Plate

- Focal length of both depends upon λ and hence both show chromatic aberration

Differences:

- A convex lens has single focal length, but a zone plate has a no of focal lengths, the image due to a convex lens is more intense than image due to a zone plate
- In case of zone plate the image is formed by diffraction
- In a convex lens all rays from the object travel the same distance and meet at the image in same phase. In a zone plate the rays from the object have different distance before meeting the image.
- For a convex lens $f_v < f_R$ while for zone plate $f_R < f_v$