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ELECTRIC POTENTIAL DUE TO A CHARGED SPHERICAL SHELL

(విద్యుదావేశిత గోళాకార కర్పరం వల్ల విద్యుత్ పొటన్షియల్)

In this topic, three cases are possible for consideration

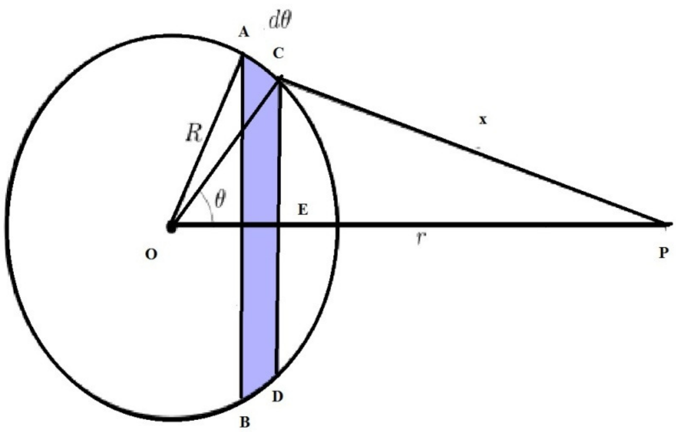
Case I: At a point outside the charged spherical shell

Case II: At a point on the surface of the spherical shell

Case III: At a point inside the charged spherical shell

Case I: At a point P lies outside the charged spherical shell (గోళాకార కర్పర బాహ్య బిందువు వద్ద)

Consider a conducting charged sphere of radius R, in which a charge q is uniformly distributed over its surface as shown in the following sphere (no charge is inside the sphere).



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The aim of this article is to derive an expression for potential at an outside point P at a distance of r from the centre of the sphere. For this purpose, the sphere is divided into number if rings with centres on OP. Further, consider one such ring ABCD as shown in the figure. Let CP=x, ∟COP=θ, and ∟AOC=dθ. From the right angled triangle OEC, CE=OC sinθ=Rsinθ

From sector AOC, AC=Rdθ.

The circumference of the ring is given by2π(Rsinθ)

Area of the ring is given by $2\pi R\sin\theta X R d\theta = 2\pi R^2 \sin\theta d\theta$

Therefore, the charge on the ring =area of the ring x surface charge density= $2\pi R^2 \sin\theta d\theta x \sigma$

Where, $\sigma = \frac{total\ charge\ on\ the\ shell}{total\ surface\ area} = \frac{q}{4\pi R^2}$

The charge dq on the ring is given by $dq = 2\pi R^2 sin\theta d\theta \times \frac{q}{4\pi R^2} = \frac{qsin\theta d\theta}{2}$ **-------**(1)

The potential at P due to the charge dq on the ring is given by

$dV = \frac{1}{4\pi\varepsilon_0} \times \frac{dq}{x} = \frac{1}{4\pi\varepsilon_0} \times \frac{qsin\theta d\theta}{2r} = \frac{qsin\theta d\theta}{8\pi\varepsilon_0 x}$ (using equation (1)) **-------**(2)

From the figure $x^2=R^2+r^2-2Rr\cos\theta$. On differentiation, one can get

$2xdx = 2Rr\sin\theta d\theta \Longrightarrow \sin\theta = \frac{xdx}{Rr}$ **-------**(3)

Substituting the value of sinθdθ from equation (3) in equation (2), we get

$dV = \frac{qxdx}{8\pi\varepsilon_0 xRr} = \frac{qdx}{8\pi\varepsilon_0 Rr}$ **------**(4)

To obtain the potential due to the whole spherical shell, we integrate equation (4) between the limits r-R and r+R

Therefore, $V = \int_{r-R}^{r+R} dV = \int_{r-R}^{r+R} \frac{qdx}{8\pi\varepsilon_0 Rr} = \frac{q}{8\pi\varepsilon_0 Rr} \int_{r-R}^{r+R} dx = \frac{q}{8\pi\varepsilon_0 Rr} [x]_{r-R}^{r+R}$

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$V = \frac{q}{8\pi\varepsilon_0 Rr}[r + R - (r - R)] = \frac{q}{8\pi\varepsilon_0 Rr}[r + R - r + R] = \frac{q}{8\pi\varepsilon_0 Rr} \times 2R \Rightarrow V = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r}$ --(5)

Equation (5) gives an expression for potential due to a charged spherical shell at an outside point.

Case II: At a point on the surface of the spherical shell(గోళాకార కర్పర తలం పై)

When the point 'P' lies on the surface, r=R

$\therefore V = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{R}$ -----(6)

Case III: At a point inside the charged spherical shell(గోళాకార కర్పర లోపలి బిందువు వద్ద)

When the point 'P' lies inside the shell, the limits of integration becomes, x=R-r and R+r

$\therefore V = \int_{R-r}^{R+r} dV = \int_{R-r}^{R+r} \frac{q dx}{8\pi\varepsilon_0 Rr} = \frac{q}{8\pi\varepsilon_0 Rr}\int_{R-r}^{R+r} dx = \frac{q}{8\pi\varepsilon_0 Rr}[x]_{R-r}^{R+r}$

$V = \frac{q}{8\pi\varepsilon_0 Rr}[R + r - R + r] = \frac{q}{8\pi\varepsilon_0 Rr}[R + r - (R - r)]\frac{q}{8\pi\varepsilon_0 Rr} \times 2r \Rightarrow V = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{R}$ -----(7)

From equations (6) and (7), one can conclude that the potential at an internal point is same as on the surface.

From equations (5), (6) and (7), the variation of potential with distance appears as follows…

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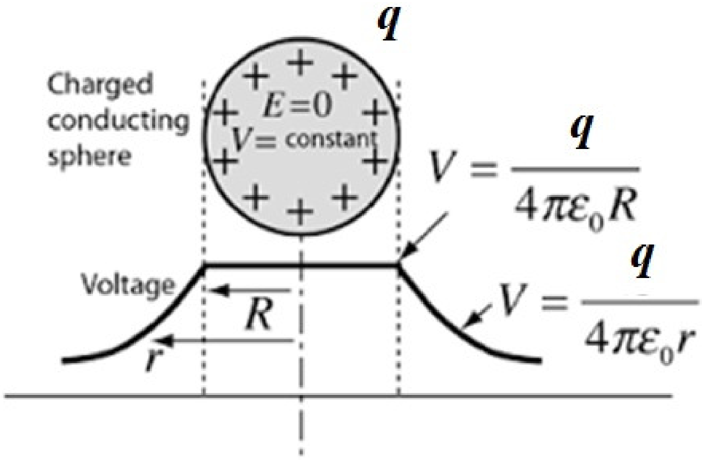

Questions

1. Define electric potential. Derive expressions for electric field due to a spherical conductor
2. Derive expressions for electric potential due to a charged spherical shell at 1) an outside point 2) a point on the surface 3) an inside point

References

1. Unified Physics, Volume III, JAI PRAKASH NATH PUBLICATIONS, MEERUT

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